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A_{ed}^V for all combinations of beam and target polarizations

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The simple expression for the deuteron vector asymmetry

$$A_{ed}^V = \frac{\varepsilon}{f P_b P_t}$$
$$\varepsilon = \frac{L-R}{L+R} \tag{1}$$

where L, R are charge and dead time normalized counts observed for left and right beam helicity respectively, f is the dilution factor and P_b, P_t are the beam and target polarizations respectively, is valid only under very restrictive conditions, that include having no contributions from other deuteron asymmetries (such as A_e, A_d^V, A_d^T, etc.), a single orientation of the target polarization, symmetric detector acceptance over all angles relative to the quantization axis, etc. Since these conditions are seldom realized in an experimental setup, it is necessary to derive a new expression that takes into account those and other factors, such as the need to combine many runs, each with different values of the polarizations h_i^+, h_i^-, $P_{1i}^+, P_{1i}^-, P_{2i}^+, P_{2i}^-$, the charges $Q_i^{+L}, Q_i^{-L}, Q_i^{+R}, Q_i^{-R}$ and dead time corrections $d_i^{+L}, d_i^{-L}, d_i^{+R}, d_i^{-R}$, which change from run to run. Here the symbols $+, -$ refer to positive and negative target polarization, and the L, R labels refer to beam helicity. h_i^+ is the helicity for runs with positive target polarization, etc. Left helicity and right helicity for the same run differ just by a minus sign.

We start with the numbers of counts for a given combination of beam (e.g. h_i^+) and target polarizations (e.g. P_{1i}^+) for a simple target made of only nitrogen and deuterium (the nitrogen represents all the unpolarized nuclei; corrections for nitrogen polarization need a special treatment, not given here.)

$$\begin{aligned}
L_i^+ &= Q_i^{+L} d_i^{+L} \Delta E' \Delta\Omega (N_N \sigma_N + N_D \sigma_{Di}^{+L}) \\
R_i^+ &= Q_i^{+R} d_i^{+R} \Delta E' \Delta\Omega (N_N \sigma_N + N_D \sigma_{Di}^{+R}) \\
L_i^- &= Q_i^{-L} d_i^{-L} \Delta E' \Delta\Omega (N_N \sigma_N + N_D \sigma_{Di}^{-L}) \\
R_i^- &= Q_i^{-R} d_i^{-R} \Delta E' \Delta\Omega (N_N \sigma_N + N_D \sigma_{Di}^{-R})
\end{aligned} \tag{2}$$

where the subindex i runs over all runs in the experiment.[1] $\Delta E' \Delta\Omega$ rep-

[1] Not all terms in the series need to be present. For example, for runs that had only positive target polarization P_{1i}^+, P_{2i}^+, the corresponding terms L_i^-, R_i^- would be zero, etc.

resent the detector acceptances, which are common to all runs for a given target and cancel out in the asymmetry, so they will be ignored from now on. N_A, $A = N$, ${}^2H(\equiv D)$, etc. are the numbers of scattering centers of each kind in the target, and their corresponding $\sigma_A(e, e'n)$ cross sections are $\sigma_N, \sigma_{Di}^{+L}, \sigma_{Di}^{+R}, \sigma_{Di}^{-L}, \sigma_{Di}^{-R}$, where the subindex i reflects the fact that the polarized deuteron cross section

$$\sigma_D = \sigma_D^0(1 + hA_e + P_1 A_d^V + P_2 A_d^T + hP_1 A_{ed}^V + hP_2 A_{ed}^T) \tag{3}$$

depends on beam and target polarizations that generally have run-dependent values. Here $\sigma_D^0 = \sigma_T + \sigma_L + \sigma_{LT} + \sigma_{TT}$ is the unpolarized ${}^2H(e, e'n)$ cross section.

In order to simplify the analysis, it is convenient to be able to factorize as many common terms as possible. If the dead time corrections $d_i^{(+/-)(L/R)}$ were equal or absent, it would be useful to express the charges as $Q_i^{-L} = \beta Q_i^{+L}$, provided $Q_i^{+L} \cong Q_i^{+R}$, etc. This is not the case, and expressing the d_i's in terms of ratios to one another does not lead to further simplification, so we are forced to keep all factors. The best approach is then, to convert the counts to charge and dead time normalized quantities, on a run by run basis, so on the r.h.s. we only have sums of $N_A \sigma_A$ products

$$\begin{aligned}
\frac{L_i^+}{Q_i^{+L} d_i^{+L}} &= N_N \sigma_N + N_D \sigma_{Di}^{+L} \\
\frac{R_i^+}{Q_i^{+R} d_i^{+R}} &= N_N \sigma_N + N_D \sigma_{Di}^{+R} \\
\frac{L_i^-}{Q_i^{-L} d_i^{-L}} &= N_N \sigma_N + N_D \sigma_{Di}^{-L} \\
\frac{R_i^-}{Q_i^{-R} d_i^{-R}} &= N_N \sigma_N + N_D \sigma_{Di}^{-R}
\end{aligned} \tag{4}$$

and the corresponding cross sections are

$$\begin{aligned}
\sigma_{Di}^{+L} &= \sigma_D^0(1 + h_i^+ A_e + P_{1i}^+ A_d^V + P_{2i}^+ A_d^T + h_i^+ P_{1i}^+ A_{ed}^V + h_i^+ P_{2i}^+ A_{ed}^T) \\
\sigma_{Di}^{+R} &= \sigma_D^0(1 - h_i^+ A_e + P_{1i}^+ A_d^V + P_{2i}^+ A_d^T - h_i^+ P_{1i}^+ A_{ed}^V - h_i^+ P_{2i}^+ A_{ed}^T) \\
\sigma_{Di}^{-L} &= \sigma_D^0(1 + h_i^- A_e - P_{1i}^- A_d^V + P_{2i}^- A_d^T - h_i^- P_{1i}^- A_{ed}^V + h_i^- P_{2i}^- A_{ed}^T) \\
\sigma_{Di}^{-R} &= \sigma_D^0(1 - h_i^- A_e - P_{1i}^- A_d^V + P_{2i}^- A_d^T + h_i^- P_{1i}^- A_{ed}^V - h_i^- P_{2i}^- A_{ed}^T)
\end{aligned} \tag{5}$$

where use has been made of the facts that

- the beam helicity changes from $+$ to $-$ for L and R counts, respectively, and that during a given run the magnitude for both signs of the helicity is the same;

• the vector target polarizations P_{1i}^- are negative relative to P_{1i}^+, although their magnitudes need not be equal;

• the tensor target polarizations P_{2i}^+, P_{2i}^- are always positive.

Arenhövel's [1] prescription to isolate A_{ed}^V requires forming a counts asymmetry that combines both signs of the beam target polarization in a difference of differences[2]

$$\varepsilon = \frac{L^+ - R^+ - (L^- - R^-)}{L^+ + R^+ + L^- + R^-}. \tag{6}$$

Forming weighted sums over the entire set of runs $NR = NR^+ + NR^-$, where the superscripts refer to the numbers of runs for each sign of the target vector polarization $P_1^{+/-}$,

$$L^+ - R^+ = \sum_{i=1}^{NR} \Big(\frac{L_i^+}{Q_i^{+L} d_i^{+L}} - \frac{R_i^+}{Q_i^{+R} d_i^{+R}} \Big) w_i^+,$$

$$L^+ + R^+ = \sum_{i=1}^{NR} \Big(\frac{L_i^+}{Q_i^{+L} d_i^{+L}} + \frac{R_i^+}{Q_i^{+R} d_i^{+R}} \Big) W_i^+, \quad \text{etc.}$$

The weights w_i for the differences are different from those for the sums, W_i, as discussed in the Appendix.

In terms of the right hand side of (4) the numerator is then

$$N_D \Big(\sum \sigma_{Di}^{+L} w_i^+ - \sum \sigma_{Di}^{+R} w_i^+ - (\sum \sigma_{Di}^{-L} w_i^- - \sum \sigma_{Di}^{-R} w_i^-) \Big) =$$
$$2N_D \sigma_D^0 \Big(\sum (h_i^+ w_i^+ - h_i^- w_i^-) A_e + \sum (h_i^+ P_{1i}^+ w_i^+ + h_i^- P_{1i}^- w_i^-) A_{ed}^V +$$
$$\sum (h_i^+ P_{2i}^+ w_i^+ - h_i^- P_{2i}^- w_i^-) A_{ed}^T \Big), \tag{7}$$

where use has been made of the fact that the terms containing A_d^V, A_d^T cancel in the difference of counts for each run, since they have the same target polarizations P_{1i}^+, P_{2i}^+ or P_{1i}^-, P_{2i}^-. The numerator can be rewritten as

$$2N_D \sigma_D^0 \Big(S_h A_e + S_{hP1} A_{ed}^V + S_{hP2} A_{ed}^T \Big), \tag{8}$$

with a new notation for the coefficients of A_e, etc.

[2] This counts asymmetry cannot be formed on a run by run basis, as discussed in the Appendix.

The denominator is

$$
\begin{gathered}
2\sum N_N\sigma_N(W_i^+ + W_i^-)+ \\
N_D\Big(\sum\sigma_{Di}^{+L}W_i^+ + \sum\sigma_{Di}^{+R}W_i^+ + \sum\sigma_{Di}^{-L}W_i^- + \sum\sigma_{Di}^{-R}W_i^-\Big) = \\
2\Big[N_N\sigma_N(\sum(W_i^+ + W_i^-) + N_D\sigma_D^0\Big(\sum(W_i^+ + W_i^-)+ \\
\sum(P_{1i}^+W_i^+ + P_{1i}^-W_i^-)A_d^V + \sum(P_{2i}^+W_i^+ + P_{2i}^-W_i^-)A_d^T\Big)\Big],
\end{gathered} \tag{9}
$$

where the coefficients for A_d^V, A_d^T have a factor of 2 that has been collected as a common factor outside the parentheses, and that cancels with the corresponding factor in the numerator.

The denominator becomes

$$
N_N\sigma_N S_W + N_D\sigma_D^0\Big(S_W + S_{P1}A_d^V + S_{P2}A_d^T\Big), \tag{10}
$$

using similar replacements for the sums as those used for the numerator. Dividing out the coefficient of $N_D\sigma_D^0$ we get

$$
\Big(\frac{N_N\sigma_N S_W}{S_W + S_{P1}A_d^V + S_{P2}A_d^T} + N_D\sigma_D^0\Big)\Big(S_W + S_{P1}A_d^V + S_{P2}A_d^T\Big). \tag{11}
$$

The complete r.h.s now reads

$$
\varepsilon = \frac{N_D\sigma_D^0}{\dfrac{N_N\sigma_N S_W}{S_W + S_{P1}A_d^V + S_{P2}A_d^T} + N_D\sigma_D^0}\,\frac{S_hA_e + S_{hP1}A_{ed}^V + S_{hP2}A_{ed}^T}{S_W + S_{P1}A_d^V + S_{P2}A_d^T}. \tag{12}
$$

The first fraction can be identified as the dilution factor

$$
f = \frac{N_D\sigma_D^0}{\dfrac{N_N\sigma_N S_W}{S_W + S_{P1}A_d^V + S_{P2}A_d^T} + N_D\sigma_D^0}. \tag{13}
$$

Dividing the numerator and denominator of (12) by S_W, and relabeling $S_h/S_W =< h >$, $S_{hp1}/S_W =< hP_1 >$, etc. we get

$$
\varepsilon = f\frac{< h > A_e + < hP_1 > A_{ed}^V + < hP_2 > A_{ed}^T}{1 + < P_1 > A_d^V + < P_2 > A_d^T}, \tag{14}
$$

with the dilution factor

$$
f = \frac{N_D\sigma_D^0}{\dfrac{N_N\sigma_N}{1 + < P_1 > A_d^V + < P_2 > A_d^T} + N_D\sigma_D^0}. \tag{15}
$$

There are several issues that must be kept in mind:

- First and foremost is finding the right weights that would reduce this method to the conventional run by run asymmetry calculation. This is probably possible only in the simplest of cases, for example when only A_{ed}^V and A_d^T contribute. In this case the weights $w_i^{+(-)}$ obtained in the Appendix get an extra factor $1/(1+P_{2i}^{+(-)}A_d^T)$, which is squared for W_i^+, W_i^-.[3] Otherwise, the problem may become intractable. Arguments regarding the importance of the remaining asymmetries A_e, etc. follow.

- The necessary condition of (14) reducing to (1) for $A_e = A_{ed}^T = A_d^V = A_d^T = 0$ is clearly satisfied. Moreover, f then reduces to the conventional form $f = N_D\sigma_D^0/(\sum N_A\sigma_A)$. It is important to note also that if the averages of the beam and target polarization were equal for both target polarization orientations, the coefficients of all asymmetries would be zero, except for A_{ed}^V and A_d^T (see below). Inspection of (5) shows this to be true, if both orientations of the target polarization were available on a run by run basis.

- For detector acceptances that are symmetric about the quantization axis, the A_e term integrates to zero. This is not the case for the proton with the TJNAF E93-026 detector, but it can be eliminated by a cut that is symmetric about the origin of the detector's x (vertical) coordinate for protons. On the other hand the combination $< P_2 > A_d^T$ never cancels out, and it is a dilution of the desired A_{ed}^V: it is a small but non-negligible model dependent correction to the asymmetry and to the dilution factor. To completely eliminate A_d^V, A_d^T a dedicated measurement of the unpolarized cross section σ_D^0 is required, taking data with the beam and target polarization off. Then A_{ed}^V can be formed exactly as prescribed by eq. (94) of ref.[1]. The term A_{ed}^T in the numerator can be neglected only if this asymmetry is known to be small, a totally model dependent assumption.

- Averaging over the number of runs takes care of unequal numbers of runs for the two target polarization orientations. Therefore, the results of a previous derivation of ε, which assumed that both signs of the target polarization were present on a run by run basis, are superseded by the present TN.

- The dilution factor changes depending on target cell. This means that essentially there is a run by run dependence of the N_A themselves. This dependence can be included in the present analysis by noting that the unpolarized contributions to the counts show up only in the denominator of

[3] Note that for this case, the r.h.s of (14) is just $f < hP_1 > A_{ed}^V/(1+ < P_2 >)A_d^T$ and by rearranging terms in the sum $S_W + S_{P2}A_d^T$ (see eq. (9)) $< hP_1 >$ cancels out, leaving just A_{ed}^V.

ε, and that they can be factored out in the dilution factor. Considering only one extra species, He (additional target components can be added in the same form), we recall that the run dependence of the dilution factor comes only from the different packing fractions p_f associated with different target cells. p_f is present in both the numerator and denominator of f, so it can be divided out of the terms for nitrogen and deuterium, leaving only N_{Hei} (and others) as terms dependent on run index. N_{Hei} can be written as

$$N_{Hei} = N_{He} \sum_j P_{fij}, \tag{16}$$

where the new index j refers to the different cells. P_f is some suitable function of p_f. Since the same cell with a given p_f could have been used for both $+$ and $-$ target polarization runs, there is no need to index P_f with $+/-$ superscripts. Then the counts from He contribute to the dilution factor as

$$\begin{aligned} 2\sigma_{He} \sum_i^{NR} N_{Hei}(W_i^+ + W_i^-) = \\ 2N_{He}\sigma_{He}(\sum_i \sum_j P_{fij}(W_i^+ + W_i^-)). \end{aligned} \tag{17}$$

Therefore, the dilution factor, including He is

$$f = \frac{N_D \sigma_D^0}{\dfrac{N_N \sigma_N + N_{He}\sigma_{He} < P_f >}{1 + < P_1 > A_d^V + < P_2 > A_d^T} + N_D \sigma_D^0}, \tag{18}$$

where $< P_f > = \sum_i \sum_j P_{fij}(W_i^+ + W_i^-))/S_W$. Keep in mind that $P_f \neq p_f$, but some function of it.

- The correction for the nitrogen asymmetry is left as an exercise for the analysis team. It better be small.

Appendix

The counts asymmetry on a run by run basis for all combinations of beam and target helicity would be

$$\varepsilon_i = \frac{\dfrac{L_i^+}{Q_i^{+L} d_i^{+L}} - \dfrac{R_i^+}{Q_i^{+R} d_i^{+R}} - \left(\dfrac{L_i^-}{Q_i^{-L} d_i^{-L}} - \dfrac{R_i^-}{Q_i^{-R} d_i^{-R}}\right)}{\dfrac{L_i^+}{Q_i^{+L} d_i^{+L}} + \dfrac{R_i^+}{Q_i^{+R} d_i^{+R}} + \dfrac{L_i^-}{Q_i^{-L} d_i^{-L}} + \dfrac{R_i^-}{Q_i^{-R} d_i^{-R}}}. \tag{19}$$

However, only the terms with either + or − target polarization are present during a given run. Moreover, when the asymmetry is computed for every run, the weighted average asymmetry for the experiment is (for each target polarization separately)

$$
\begin{aligned}
< A >= & \qquad \frac{\sum A_i/\delta^2 A_i}{\sum 1/\delta^2 A_i} \\
A_i = \frac{\varepsilon_i}{f h_i P_i}, & \qquad \delta^2 A_i \cong \frac{1}{(f h_i P_i)^2 N_i},
\end{aligned}
\tag{20}
$$

where A is a generic asymmetry. With $\varepsilon_i = (L_i - R_i)/(L_i + R_i) = \Delta_i/N_i$ the average A effectively is

$$
< A >= \frac{\sum \Delta_i f h_i P_i}{\sum (f h_i P_i)^2 N_i} = \frac{1}{f} \frac{\sum h_i P_i \Delta_i}{\sum (h_i P_i)^2 N_i} \tag{21}
$$

Since the ratio of the sums is not in general equal to the sum of the ratios, we don't expect that forming an overall counts asymmetry from sums running over the number of runs will lead to the above result, unless the separate sums $\sum(L_i - R_i)$ and $\sum(L_i + R_i)$ are properly weighted

$$
\varepsilon = \frac{\sum \Delta_i w_i}{\sum N_i W_i} = \frac{\sum N_D \sigma_D^0 h_i P_i A w_i}{\sum (N_N \sigma_N + N_D \sigma_D^0) W_i} = f A \frac{\sum h_i P_i w_i}{\sum W_i}. \tag{22}
$$

Eq. (22) agrees with eq. (21) if the weights are $w_i = h_i P_i$ and $W_i = (h_i P_i)^2$.

References

[1] H. Arenhövel, W. Leidemann and E.L. Tomusiak, Z. Phys. A, 331, 123 (1988)