I. NUCLEON ELECTRIC FORM FACTOR MEASUREMENT BY CROSS SECTION DIFFERENCE METHOD

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The difference of cross sections for polarized electron-polarized nucleon scattering in the region of the quasi elastic peak is given by eq. (4) in the proposal. For the orientation of the target magnetic field perpendicular to the momentum transfer ($\theta^* = \pi/2, \phi^* = 0$) we have:

$$\Delta = -2\sigma_{Mott}\frac{E'}{E_0}\sqrt{\frac{\tau}{1+\tau}}\tan\frac{\theta_e}{2}\{G_E^{p,n}G_M^{p,n}\} \tag{1}$$

where σ_{Mott} is the Mott cross section, E_0 and E' are the beam and scattered electron energies, θ_e is the electron scattering angle, and $\tau = Q^2/4M^2$ depends on the four momentum transfer $Q^2 = -q^2$ and the nucleon mass M. The kinematic dependence of Δ at fixed τ ($\equiv$ fixed Q^2) is dominated by the Mott cross section, which is largest at small angles. Δ is the beam helicity dependent part of the cross section

$$\sigma(h) = \Sigma + h\Delta. \tag{2}$$

The rates $L(R)$ for two opposite orientations of the beam helicity h and target polarization are proportional to the polarized cross sections $\sigma_{L(R)}$

$$L(R) = \sigma_{L(R)}\mathcal{L}_{L(R)}\mathcal{A} \tag{3}$$

where $\mathcal{L}_{\mathcal{L}(\mathcal{R})}$ represents the luminosities $\mathcal{L}_{L(R)} = I_{L(R)}z$, with $I_{L(R)}$ as the beam currents for $L(R)$ beam helicities and z is the target thickness. $\mathcal{A}$ represents the detector acceptances $\mathcal{A} = \Delta\Omega_e\Delta\Omega_N\Delta E'$, where $\Delta\Omega_{e,N}$ are the HMS and nucleon detector solid angles, respectively. The polarized cross sections depend on the beam and target polarizations P_b, P_t. For deuterium targets we have:

$$\sigma_{L(R)} = \Sigma \pm P_bP_t\Delta_m = \Sigma(1 \pm P_bP_tA_{eD}^V) \tag{4}$$

where $A_{eD}^V = \gamma A_{p,n}$ or $\Delta_m = \gamma\Delta$ and γ is the deuteron D-state correction. For ordinary ammonia targets $\Delta_m = \Delta$.

The numbers of counts $N_{L(R)}$ over a time interval Δt are proportional to the rates corrected for the detector efficiencies $N_{L(R)} = \eta L(R)\Delta t$. The complete expression is then

$$N_{L(R)} = Q_{L(R)}z\eta\Delta\Omega_e\Delta\Omega_N\Delta E'(\Sigma \pm P_bP_t\gamma\Delta) \tag{5}$$

where we have substituted the total charge $Q = I\Delta t$. From the difference in counts $N_L - N_R$ we can solve for the helicity dependent part of the cross section:

$$\Delta = \frac{2}{z\eta\gamma P_bP_t\Delta\Omega_e\Delta\Omega_N\Delta E'}\Big(\frac{N_L}{Q_L} - \frac{N_R}{Q_R}\Big) \tag{6}$$

Notice the absence of the dilution factor f in this expression. This is because the contributions of the unpolarized materials to the counting rates, which depend only on the unpolarized cross section Σ cancel in the difference.

The relative uncertainties in the measured cross section are the uncorrelated sum of uncertainties in the corresponding factors in eq. (6) above. Using conservative values for the errors in each quantity we obtain:

Relative errors

Source		Relative error %		Comments
		Inclusive	Exclusive	
Statistics	N	2	1.5 - 5.7	This is Q^2 dependent, it is smallest at 0.5 GeV^2
Charge	Q	2	2	
Target thickness	z	4	4	Hard to beat down, but possible
Detector efficiency	η	2	2	? for proton detection
D-state correction	γ	0	1	Inclusive measurement on NH_3
HMS solid angle	$\Delta\Omega_e$	5	5	
Nucleon detector solid angle	$\Delta\Omega_N$	0	5	
Final electron energy	E'	1	1	
Beam polarization	P_b	2	2	
Target polarization	P_t	2.5	4	2.5 for NH3, 4 for ND_3

Combining the errors in quadrature the total is $\simeq$ 8% for the inclusive case and between 10% and 11.3% for the exclusive case, both very competitive at any momentum transfer. The main sources are the statistical uncertainty, the target thickness (for which the packing fraction needs to be known, from the ratio of rates to the carbon target) and the detector acceptances. There are no contributions from the dilution factor (not to be confused with the packing fraction; the latter is the effective thickness of the ND_3 in [cm], not the number of polarizable nucleons).

In a worst case scenario, the relative errors in the charge and detector efficiency could be as large as 5%. Although the latter is unlikely to be so poor for proton detection, the resulting combined errors for the inclusive case would be 9.2% and would range from 10.9% to 12.1% for the exclusive case.