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**Spin Asymmetries $A_1$ and $A_2$ and Measured Asymmetries $A_\parallel$ and $A_\perp$**

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# 1 Introduction

We need to convert the measured asymmetries $A_\parallel$ and $A_\perp$ into the physics quantities of interest, the spin asymmetries $A_1$ and $A_2$. The conventional expressions linking the two pairs of quantities (eq. (10)) apply only when the scattering is in the same plane that contains the direction of the nucleon spin, which is determined by the polarized target magnetic field. This approach is valid when the final energy $E'$ of the leptons is large, so that the deflections caused by the target field are small, and the components in the plane are very similar to the total magnitudes.

However, in *RSS* the final electron energies are as low as 3.4 GeV, which, when the field is in the transverse or normal to the beam orientation, result in deflections of up to 3.6° along the particles' trajectories. These deflections translate into tilts of the scattering plane relative to the horizontal laboratory plane of up to 16°, that need to be included in the extraction of $A_1$ and $A_2$.

A summary of the derivation and the resulting equations plus numerical examples are given below.

# 2 Definitions

The starting point is the expression for the difference of cross sections for inclusive inelastic polarized leptons scattering on polarized nuclei, for opposite relative alignments of the target spin relative to the beam helicity [1, 2, 3]

$$\Delta\sigma = \sigma^{\uparrow\uparrow} - \sigma^{\uparrow\downarrow} =$$

$$\frac{-4\alpha^2}{Q^2}\frac{E'}{E}\Big[(E\cos\theta_N + E'\cos\alpha)MG_1 + 2EE'(\cos\alpha - \cos\theta_N)G_2\Big] \qquad (1)$$

where $\sigma^{\uparrow\uparrow(\uparrow\downarrow)} = d^2\sigma^{\uparrow\uparrow(\uparrow\downarrow)}/(d\Omega dE')$ is the inclusive double differential cross section in the laboratory frame[1]; $\Omega$ is the solid angle element in this frame; $\alpha$ is the fine structure constant (defined explicitly here to avoid confusion with the angle $\alpha$); $Q^2$ is the squared four-momentum transfer; $E$ and $E'$ are the beam and scattered lepton energies; $M$ is the nucleon mass; and $G_1(Q^2, W)$ and $G_2(Q^2, W)$ are the nucleon spin structure functions of $Q^2$ and of the final state's invariant mass $W$. In the scaling limit $Q^2$, $\nu \to \infty$, $G_1$ and $G_2$ reduce to $M^2\nu G_1 = g_1(x)$, $M\nu^2 G_2 = g_2(x)$, where $\nu = E - E'$ is the virtual photon energy loss and $x = Q^2/(2M\nu)$ is the Bjorken scaling variable.

The angle $\theta_N$ is the polar angle of the aligned nuclear spin direction $\mathbf{S}$ in a right handed coordinate system with the $z$ axis along the beam direction, and the $y$ axis pointing along the vector product of the beam and scattered electron momenta $\mathbf{k}\times\mathbf{k'}$. The plane containing $\mathbf{k}$ and $\mathbf{k'}$ defines the scattering plane $(x, z)$. The azimuthal angle $\phi$ of the nuclear spin vector in this system can also be viewed as the tilt of the scattering plane in a system in which the $(x, z)$ plane is defined by $\mathbf{k}$ and the nuclear spin (i.e. the horizontal plane in the lab). The angle $\alpha$ is defined in terms of the scattering angle $\theta = \arccos(\mathbf{k}\cdot\mathbf{k'}/(kk'))$, $\theta_N$ and $\phi$

$$\cos\alpha = \sin\theta_N \sin\theta \cos\phi + \cos\theta_N \cos\theta. \tag{2}$$

Ref. [3] gives a detailed derivation of the angle $\alpha$.

The unpolarized cross section can be written in a convenient form as

$$\begin{aligned} \sigma = \frac{d^2\sigma}{d\Omega dE'} = & \quad \Gamma_T(\sigma_T + \varepsilon\sigma_L) \\ = & \quad \sigma_{Mott}\left(W_2 + 2\tan^2(\frac{\theta}{2})W_1\right) \\ = & \quad \frac{2\alpha^2}{Q^2}\,\frac{E'}{E}\left(W_1 + \frac{W_2}{2\tan^2(\theta/2)}\right), \end{aligned} \tag{3}$$

where $\sigma_L$ and $\sigma_T$ are the longitudinal and transverse virtual photon absorption cross sections; $\varepsilon$ is the degree of longitudinal polarization of the virtual photon, and $\Gamma_T$ is the flux of virtual photons

$$\varepsilon = \frac{1}{1 + 2(1 + \nu^2/Q^2)\tan^2(\theta/2)},$$
$$\Gamma_T = \frac{\alpha}{2\pi^2 Q^2}\frac{W^2 - M^2}{2M}\frac{E'}{E}\frac{1}{1-\varepsilon};$$

[1]Note that authors use different notations when referring to this cross section, in terms of other kinematic variables.

and $\sigma_{Mott} = [\alpha \cos(\theta/2)/(2E\sin^2(\theta/2))]^2$. $W_1(Q^2,W)$ and $W_2(Q^2,W)$ are the unpolarized lepton-nucleon structure functions. In the scaling limit $W_1$ and $W_2$ reduce to $F_1(x) = MW_1$ and $F_2(x) = \nu W_2$. $W_1$ and $W_2$ are related by

$$\frac{W_2}{W_1} = \frac{1 + R(Q^2,W)}{1 + \nu^2/Q^2}, \tag{4}$$

where $R(Q^2,W) = \sigma_L/\sigma_T$ is another unpolarized structure function.

# 3 Asymmetries

With the above definitions the cross section differences for the (anti)longitudinal[2] (**-S**$\|$**k**) and transverse (**S**$\perp$**k**) configurations of the field relative to the beam direction can be written as

$$\begin{aligned} \Delta\sigma_{\|} &= \frac{4\alpha^2}{Q^2}\frac{E'}{E}((E + E'\cos\theta)MG_1 - Q^2G_2) \\ \Delta\sigma_{\perp} &= \frac{4\alpha^2}{Q^2}\frac{E'}{E}E'\sin\theta\cos\phi(MG_1 + 2EG_2) \end{aligned} \tag{5}$$

which are obtained by substituting in eq. (1), $\theta_N = 180°$ for the longitudinal case and $\theta_N = -90°$ for the transverse one.

The measured $A_{\|}$ and $A_{\perp}$ can be constructed from the respective cross section differences divided by the sum, which is just twice the unpolarized cross sections. We make use of the convenient cancellations of factors among these equations and the last form of eq. (3), to get

$$\begin{aligned} A_{\|} &= \frac{\Delta\sigma_{\|}}{2\sigma} = \frac{D'}{W_1}((E + E'\cos\theta)MG_1 - Q^2G_2) \\ A_{\perp} &= \frac{\Delta\sigma_{\perp}}{2\sigma} = \frac{D'}{W_1}E'\sin\theta\cos\phi(MG_1 + 2EG_2) \\ D' &= \frac{1-\varepsilon}{1+\varepsilon R}. \end{aligned} \tag{6}$$

[2]During *RSS* the target chamber was rotated clockwise from its orientation during TJ-NAF E93-026 ($G_E^n$) to align the magnet axis with the beam. The magnet axis pointed to the left of the beam during E93-026 to deflect the beam downwards towards the beam dump and, since the magnet power supply leads were not reversed, the field pointed upstream of the beam during *RSS*. The corresponding direction of the field for the transverse configuration was -90° ($+x$ axis points to the right of the beam).

This system of equations needs to be solved for $G_1$ and $G_2$. The solutions are

$$
\begin{aligned}
\frac{MG_1}{W_1} &= \frac{Q^2(\cot(\theta/2)\cos\phi\ A_\parallel + A_\perp)}{D'E'\sin\theta\cos\phi\ (Q^2+2E(E+E'\cos\theta))} \\
\frac{G_2}{W_1} &= \frac{-E'\sin\theta\cos\phi\ A_\parallel + (E+E'\cos\theta)A_\perp}{D'E'\sin\theta\cos\phi\ (Q^2+2E(E+E'\cos\theta))}.
\end{aligned}
\tag{7}
$$

The reason for leaving the structure functions in the l.h.s. is apparent when looking at the expressions for the spin asymmetries

$$
\begin{aligned}
A_1 &= \nu\frac{MG_1}{W_1} - Q^2\frac{G_2}{W_1} \\
A_2 &= \sqrt{Q^2}\Big(\frac{MG_1}{W_1} + \nu\frac{G_2}{W_1}\Big).
\end{aligned}
\tag{8}
$$

These expressions are derived directly from the definitions of $A_1$ and $A_2$ in terms of total photoabsorption cross sections. Substituting eq. (7) in the above expressions yields the final result

$$
\begin{aligned}
A_1 &= \frac{Q^2}{D'}\frac{(\nu\cot(\theta/2) + E'\sin\theta)\cos\phi\ A_\parallel - E'(1+\cos\theta)A_\perp}{E'\sin\theta\cos\phi\ (Q^2+2E(E+E'\cos\theta))} \\
A_2 &= \frac{\sqrt{Q^2}}{D'}\frac{(Q^2\cot(\theta/2) - \nu E'\sin\theta)\cos\phi\ A_\parallel + (Q^2+\nu(E+E'\cos\theta))A_\perp}{E'\sin\theta\cos\phi\ (Q^2+2E(E+E'\cos\theta))}
\end{aligned}
\tag{9}
$$

# 4 Numerical examples and other considerations

The expressions for $A_1$ and $A_2$ given in eq. (9), setting $\phi = 0$, can be compared numerically to the results obtained from the usual formulas [4]

$$
\begin{aligned}
A_1 &= \frac{C}{D}\Big(A_{||} - dA_\perp\Big) \\
A_2 &= \frac{C}{D}\Big(c'A_{||} + d'A_\perp\Big)
\end{aligned}
\tag{10}
$$

where $C = 1/(1+\eta c')$; $\eta = \varepsilon\sqrt{Q^2}/(E-\varepsilon E')$; $c' = \eta(1+\varepsilon)/(2\varepsilon)$; $D = (1-\varepsilon E'/E)/(1+\varepsilon R)$ is the virtual photon depolarization factor; $d' = 1/\sqrt{2\varepsilon/(1+\varepsilon)}$; and $d = \eta d'$.

Both sets of equations agree, within the precision of rounding errors, at the level of ratios of coefficients of $A_\perp$ over $A_\parallel$, and in the overall values of $A_1$ and $A_2$.

Equations (10) are usually derived for the case of $\theta_N = 0°$ and $90°$. For the case of $\theta_N = 180°$ and $-90°$ there are overall (-) signs for each. However, since the $A_\parallel$ and $A_\perp$ asymmetries measured in *RSS* are defined as having the opposite signs as the ones for 0° and 90°, there is a sign cancellation, and the results with both sets of expressions agree, as expected.

The table below gives the sign of the coefficients of $A_\parallel$ and $A_\perp$ in eq. (7) for several combinations of $\theta_N$

| Function | Asymmetry | $\theta_N$ parallel, perpendicular | | |
|---|---|---|---|---|
| | | 0°, 90° | 180°, 90° | 180°, -90° |
| $\frac{MG_1}{W_1}$ | $A_\parallel$ | $-$ | $+$ | $+$ |
| | $A_\perp$ | $-$ | $-$ | $+$ |
| $\frac{G_2}{W_1}$ | $A_\parallel$ | $+$ | $-$ | $-$ |
| | $A_\perp$ | $-$ | $-$ | $+$ |

In applying the expressions above, it must be kept in mind that the deflection of the scattered electrons by the target field is a function of the electron momentum **k'**, and therefore $\phi(W)$ is not a constant value. The effective $\phi$ for each bin in $W$ can be either calculated in the analysis engine or by hand. In the calculation by hand, some type of code for particle tracking in the target magnetic field (such as `magnet8a.f` [5]) is needed to calculate the deflection angle $\delta$ along the particle's trajectory relative to the horizontal plane. This angle in turn is converted to $\phi$ using

$$\tan\phi = \frac{\tan\delta}{\sin\theta}, \tag{11}$$

since, as indicated above, $\phi$ can be equally measured relative to the horizontal plane instead of the scattering plane.

The deflections for the $A_\perp$ configuration at 90° range from 2.67° for elastic scattering to 3.72° for $W = 1.985$ GeV. The corresponding $\phi$'s range from 11.61° to 16.01°. Since $\phi$ enters only as the argument of the cosine, the effect of even these large tilts is relatively small, a 2.5% or less correction to the asymmetries.

The small dependence of $\delta$ on the actual scattering angle $\theta$ for each event is entirely negligible, since it has a variation at fixed $W$ of less than 7% across the entire HMS $\theta$ acceptance.

## 5 Coefficients of $A_\parallel$, $A_\perp$ for ntuple

The expressions in eq. (9) can be rewritten in terms of basic kinematic quantities $E$, $E'$ and $\theta$ so that the coefficients of $A_\parallel$ and $A_\perp$ can be computed in the analyzer code and stored in the ntuple. Starting with [3]

$$\begin{aligned} A_1 = & \frac{1}{(E+E')D'}\Big((E-E'\cos\theta)A_\parallel - \frac{E'\sin\theta}{\cos\phi}A_\perp\Big) \\ A_2 = & \frac{\sqrt{Q^2}}{2ED'}\Big(A_\parallel + \frac{E-E'\cos\theta}{E'\sin\theta\cos\phi}A_\perp\Big) \end{aligned} \tag{12}$$

the coefficients that can be calculated for every $W$ bin are:

$$\begin{aligned} \texttt{na1} = & \frac{1}{E+E'} \\ \texttt{ca1par} = & E - E'\cos\theta \\ \texttt{ca1per} = & \frac{E'\sin\theta}{\cos\phi} \\ \texttt{na2} = & \frac{\sqrt{Q^2}}{2E} \\ \texttt{ca2per} = & \frac{E-E'\cos\theta}{E'\sin\theta\cos\phi} \end{aligned}$$

in addition to $\varepsilon$, which is needed to compute $D'$. $D'$ cannot be stored in the ntuple, because it is a function of the $R$ structure function, which is calculated separately.

## References


[1] C. E. Carlson and Wu-Ki Tung, Phys. Rev. D **5** 721 (1972).

[2] A. J. G. Hey and J. E. Mandula, Phys. Rev. D **5** 2610 (1972).


[3] The expressions below have been verified to agree numerically with those of eq. (9).

[3] M. Anselmino, A. Efremov and E. Leader, Phys. Rep. **262**, 1 (1995).

[4] Oscar A. Rondon, *Precision measurement of the nucleon spin structure functions in the region of the nucleon resonances*, TJNAF E-96-002, January 1997.

[5] R. Lourie, modified by O. A. Rondon, `magnet8a.f`, *A FORTRAN program to track charged particle trajectories in a magnetic field*, available on request from the author of this TN.