THEORETICAL ASPECTS OF TRANSVERSITY

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The classification of quark distribution functions in transversely polarized hadrons is presented. Time-reversal 'odd' distributions are also discussed.

I will present a classification of quark distribution functions, in particular for transversely polarized hadrons. This contribution is based to a large extent on the recent report by Barone, Ratcliffe and myself [1], where a rather huge list of references can be found. The discussion on the so-called time-reversal odd distribution functions has been expanded to take into account works which appeared very recently in the literature.

At leading-twist level there are three distribution functions if transverse momentum is neglected [2, 3]:
the number density, or unpolarised distribution $f(x)$
the longitudinal polarisation, or helicity distribution $\Delta f(x)$
the transverse polarisation, or transversity distribution $\Delta_T f(x)$

$$\begin{aligned} f(x) &= f_+(x) + f_-(x) \\ \Delta f(x) &= f_+(x) - f_-(x) \\ \Delta_T f(x) &= f_\uparrow(x) - f_\downarrow(x) \end{aligned}$$

As we will see later, if we admit a finite quark transverse momentum $\boldsymbol{k}_\perp$, five new distributions appear. Two of them are the so-called time-reversal odd distributions:

$$\overbrace{f\,,\ \Delta f\,,\ \Delta_T f}^{\text{no } \boldsymbol{k}_\perp}\,,\ \overbrace{g_{1T}\,,\ h_{1L}^\perp\,,\ h_{1T}^\perp\,,\ \underbrace{f_{1T}^\perp\,,\ h_1^\perp}_{T\text{-odd}}}^{\boldsymbol{k}_\perp\text{-dependent}}\,.$$

In the following I will discuss in detail the origin of the above outlined classification.

1. Leading-twist, $k_\perp$-independent distributions

1.1. *Correlation matrix*

It is convenient to introduce the correlation matrix:

$$\Phi_{ij}(k,P,S) = \int \mathrm{d}^4\xi\, \mathrm{e}^{\mathrm{i}k\cdot\xi}\, \langle PS|\bar{\psi}_j(0)\mathcal{L}^\dagger(0,\infty)\mathcal{L}(\infty,\xi)\psi_i(\xi)|PS\rangle \qquad (1.1.1)$$

where, to make Φ gauge invariant a link operator has been inserted between the quark fields. The link reads:

$$\mathcal{L}(y,\infty) = \mathcal{P}\, \exp\left(-\mathrm{i}g \int_y^\infty \mathrm{d}s_\mu\, A^\mu(s)\right). \qquad (1.1.2)$$

Quark distribution functions can then be expressed as

$$\mathrm{Tr}(\Gamma\Phi) = \int \mathrm{d}^4\xi\, \mathrm{e}^{\mathrm{i}k\cdot\xi}\, \langle PS|\bar{\psi}(0)\, \mathcal{L}^\dagger(0,\infty)\Gamma\, \mathcal{L}(\infty,\xi)\psi(\xi)|PS\rangle \qquad (1.1.3)$$

The presence of the link in the definition of distributions is crucial, since it is responsible for the non-vanishing of the time-reversal odd distributions [4]. We will come back to this point later.
The correlation function (1.1.1) transforms as

$$\Phi^\dagger(k,P,S) = \gamma^0\, \Phi(k,P,S)\, \gamma^0 \quad \text{(hermiticity)} \qquad (1.1.4)$$

$$\Phi(k,P,S) = \gamma^0\, \Phi(\tilde{k},\tilde{P},-\tilde{S})\, \gamma^0 \quad \text{(parity)} \qquad (1.1.5)$$

If the link operators were not inserted in the definition of the correlation function, the following relation would hold:

$$\Phi^*(k,P,S) = \gamma^5 C\, \Phi(\tilde{k},\tilde{P},\tilde{S})\, C^\dagger\gamma^5 \quad (\textit{naive}\ \text{time-reversal}). \qquad (1.1.6)$$

Actually, due to the link, the transformation under time reversal is more complicated and it will be discussed later.
The most general decomposition of Φ in a basis of Dirac matrices is

$$\Phi(k,P,S) = \tfrac{1}{2}\left\{\mathcal{S}\, \mathbb{1} + \mathcal{V}_\mu\, \gamma^\mu + \mathcal{A}_\mu\gamma^5\gamma^\mu + \mathrm{i}\mathcal{P}_5\gamma^5 + \tfrac{1}{2}\mathrm{i}\, \mathcal{T}_{\mu\nu}\, \sigma^{\mu\nu}\gamma^5\right\}. \qquad (1.1.7)$$

Imposing the constraints on hermiticity, parity and *naive* time-reversal we get:

$$\mathcal{S} = \tfrac{1}{2}\,\mathrm{Tr}(\Phi) = C_1 \tag{1.1.8a}$$

$$\mathcal{V}^\mu = \tfrac{1}{2}\,\mathrm{Tr}(\gamma^\mu\Phi) = C_2\,P^\mu + C_3\,k^\mu \tag{1.1.8b}$$

$$\mathcal{A}^\mu = \tfrac{1}{2}\,\mathrm{Tr}(\gamma^\mu\gamma_5\Phi) = C_4\,S^\mu + C_5\,k{\cdot}S\,P^\mu + C_6\,k{\cdot}S\,k^\mu \tag{1.1.8c}$$

$$\mathcal{P}_5 = \tfrac{1}{2\mathrm{i}}\,\mathrm{Tr}(\gamma_5\Phi) = 0 \tag{1.1.8d}$$

$$\mathcal{T}^{\mu\nu} = \tfrac{1}{2\mathrm{i}}\,\mathrm{Tr}(\sigma^{\mu\nu}\gamma_5\Phi) = C_7 P^{[\mu}S^{\nu]} + C_8 k^{[\mu}S^{\nu]} + C_9 k{\cdot}S P^{[\mu}k^{\nu]} \tag{1.1.8e}$$

If we relax the constraint of time-reversal invariance coming from the *naive* eq.(1.1.6) three more terms appear:

$$\mathcal{V}^\mu = \cdots + C_{10}\,\varepsilon^{\mu\nu\rho\sigma}S_\nu P_\rho k_\sigma \tag{1.1.9}$$

$$\mathcal{P}_5 = C_{11}\,k{\cdot}S \tag{1.1.10}$$

$$\mathcal{T}^{\mu\nu} = \cdots + C_{12}\,\varepsilon^{\mu\nu\rho\sigma}P_\rho k_\sigma \tag{1.1.11}$$

Leading-twist contributions are of order $\mathcal{O}(P^+)$ in the infinite momentum frame. Ignoring quark transverse momentum, only the vector, axial, and tensor terms survive:

$$\mathcal{V}^\mu = \tfrac{1}{2}\int \mathrm{d}^4\xi\,\mathrm{e}^{\mathrm{i}k\cdot\xi}\,\langle PS|\overline{\psi}(0)\gamma^\mu\psi(\xi)|PS\rangle = A_1\,P^\mu \tag{1.1.12a}$$

$$\mathcal{A}^\mu = \tfrac{1}{2}\int \mathrm{d}^4\xi\,\mathrm{e}^{\mathrm{i}k\cdot\xi}\,\langle PS|\overline{\psi}(0)\gamma^\mu\gamma^5\psi(\xi)|PS\rangle = \lambda_N\,A_2\,P^\mu \tag{1.1.12b}$$

$$\mathcal{T}^{\mu\nu} = \tfrac{1}{2\mathrm{i}}\int \mathrm{d}^4\xi\,\mathrm{e}^{\mathrm{i}k\cdot\xi}\,\langle PS|\overline{\psi}(0)\sigma^{\mu\nu}\gamma^5\psi(\xi)|PS\rangle = A_3\,P^{[\mu}S_\perp^{\nu]} \tag{1.1.12c}$$

The leading-twist distribution functions $f(x)$, $\Delta f(x)$ and $\Delta_T f(x)$ are obtained by integrating A_1, A_2 and A_3, respectively, over k, with the constraint $x = k^+/P^+$:

$$\left\{\begin{matrix} f(x) \\ \Delta f(x) \\ \Delta_T f(x) \end{matrix}\right\} = \int \frac{\mathrm{d}^4k}{(2\pi)^4}\left\{\begin{matrix} A_1(k^2, k{\cdot}P) \\ A_2(k^2, k{\cdot}P) \\ A_3(k^2, k{\cdot}P) \end{matrix}\right\}\delta\!\left(x - \frac{k^+}{P^+}\right) \tag{1.1.13}$$

that is

$$f(x) = \frac{1}{2}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\mathrm{Tr}(\gamma^+\Phi)\,\delta(k^+ - xP^+) \tag{1.1.14a}$$

$$\Delta f(x) = \frac{1}{2}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\mathrm{Tr}(\gamma^+\gamma_5\Phi)\,\delta(k^+ - xP^+) \tag{1.1.14b}$$

$$\Delta_T f(x) = \frac{1}{2}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\mathrm{Tr}(\gamma^+\gamma^1\gamma_5\Phi)\,\delta(k^+ - xP^+) \tag{1.1.14c}$$

The three leading-twist distribution functions are light-cone Fourier transforms of expectation values of quark-field bilinears:

$$f(x) = \int \frac{\mathrm{d}\xi^-}{4\pi} \mathrm{e}^{\mathrm{i}xP^+\xi^-} \langle PS|\bar{\psi}(0)\gamma^+\psi(0,\xi^-,0_\perp)|PS\rangle \tag{1.1.15a}$$

$$\Delta f(x) = \int \frac{\mathrm{d}\xi^-}{4\pi} \mathrm{e}^{\mathrm{i}xP^+\xi^-} \langle PS|\bar{\psi}(0)\gamma^+\gamma_5\psi(0,\xi^-,0_\perp)|PS\rangle \tag{1.1.15b}$$

$$\Delta_T f(x) = \int \frac{\mathrm{d}\xi^-}{4\pi} \mathrm{e}^{\mathrm{i}xP^+\xi^-} \langle PS|\bar{\psi}(0)\gamma^+\gamma^1\gamma_5\psi(0,\xi^-,0_\perp)|PS\rangle \tag{1.1.15c}$$

1.2. *Antiquark distributions*

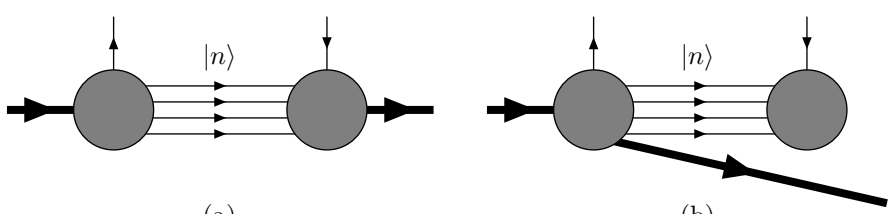
Formal relations between quark and antiquark distribution functions exist:

$$\bar{f}(x) = -f(-x) \tag{1.2.1a}$$

$$\Delta\bar{f}(x) = \Delta f(-x) \tag{1.2.1b}$$

$$\Delta_T\bar{f}(x) = -\Delta_T f(-x)\,. \tag{1.2.1c}$$

These relations are difficult to use in practice for computing the antiquark distributions, since for $x < 0$ connected *and* semiconnected diagrams contribute to the previous functions (contributions (a) and (b) in the figure) [5]. At the moment, calculations of the antiquark transversity distributions have been presented in Ref. [6] and in Ref. [7]. While in [6] these distributions have been obtained inserting 4-quarks intermediate states in the expression of antiquark distributions, in Ref. [7] eq.(1.2.1c) has been used and the previous caveat therefore applies.



(a) (b)

1.3. *Probabilistic interpretation of distribution functions*

To see more easily the physical meaning of the leading twist distributions they can be rewritten as it follows:

$$f(x) = \frac{1}{\sqrt{2}} \sum_n \delta\big((1-x)P^+ - P_n^+\big) \, |\langle PS|\psi_{(+)}(0)|n\rangle|^2 \tag{1.3.1}$$

$$\Delta f(x) = \frac{1}{\sqrt{2}} \sum_n \delta\left((1-x)P^+ - P_n^+\right) \times \left\{ \left|\langle PS|\mathcal{P}_+\psi_{(+)}(0)|n\rangle\right|^2 - \left|\langle PS|\mathcal{P}_-\psi_{(+)}(0)|n\rangle\right|^2 \right\} \tag{1.3.2}$$

$$\Delta_T f(x) = \frac{1}{\sqrt{2}} \sum_n \delta\left((1-x)P^+ - P_n^+\right) \times \left\{ \left|\langle PS|\mathcal{P}_\uparrow\psi_{(+)}(0)|n\rangle\right|^2 - \left|\langle PS|\mathcal{P}_\downarrow\psi_{(+)}(0)|n\rangle\right|^2 \right\} \tag{1.3.3}$$

where the Pauli-Lubanski projectors are:
$\mathcal{P}_\pm = \frac{1}{2}\left(1 \pm \gamma^5\right)$ (for helicity)
$\mathcal{P}_{\uparrow\downarrow} = \frac{1}{2}(1 \pm \gamma^1\gamma^5)$ (for transverse polarisation).
It is possible to define vector, axial and tensor charges as first moments of the distributions:

$$\int_{-1}^{+1} \mathrm{d}x \, f(x) = \int_0^1 \mathrm{d}x \left[f(x) - \bar{f}(x)\right] = g_V \tag{1.3.4a}$$

$$\int_{-1}^{+1} \mathrm{d}x \, \Delta f(x) = \int_0^1 \mathrm{d}x \left[\Delta f(x) + \Delta\bar{f}(x)\right] = g_A \tag{1.3.4b}$$

$$\int_{-1}^{+1} \mathrm{d}x \, \Delta_T f(x) = \int_0^1 \mathrm{d}x \left[\Delta_T f(x) - \Delta_T\bar{f}(x)\right] = g_T \tag{1.3.4c}$$

The vector and tensor charges are the first moments of flavour non-singlet combinations (quarks minus antiquarks) whereas the axial charge is the first moment of a flavour singlet combination (quarks plus antiquarks).

1.4. *Quark-nucleon helicity amplitudes*

Another way to classify leading-twist quark distribution functions comes by expressing the distributions in terms of quark-nucleon forward amplitudes. There are in general 16 amplitudes. Imposing helicity conservation,

$$\Lambda + \lambda = \Lambda' + \lambda' \tag{1.4.1}$$

only 6 amplitudes survive:

$$\mathcal{A}_{++,++}\,,\ \mathcal{A}_{--,--}\,,\ \mathcal{A}_{+-,+-}\,,\ \mathcal{A}_{-+,-+}\,,\ \mathcal{A}_{+-,-+}\,,\ \mathcal{A}_{-+,+-} \tag{1.4.2}$$

Parity invariance implies

$$\mathcal{A}_{\Lambda\lambda,\Lambda'\lambda'} = \mathcal{A}_{-\Lambda-\lambda,-\Lambda'-\lambda'} \tag{1.4.3}$$

and gives the following 3 constraints on the amplitudes:

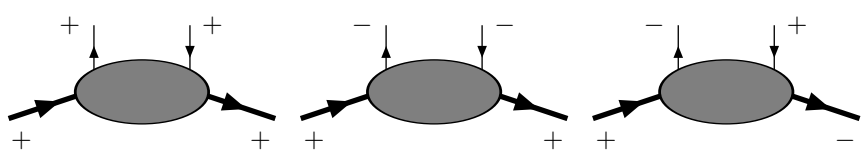
$$\mathcal{A}_{++,++} = \mathcal{A}_{--,--} \tag{1.4.4}$$

$$\mathcal{A}_{++,--} = \mathcal{A}_{--,++} \tag{1.4.5}$$

$$\mathcal{A}_{+-,-+} = \mathcal{A}_{-+,+-} \tag{1.4.6}$$

We are left with three independent amplitudes

$$\mathcal{A}_{++,++}\,,\quad \mathcal{A}_{+-,+-}\,,\quad \mathcal{A}_{+-,-+} \tag{1.4.7}$$



Two amplitudes, $\mathcal{A}_{++,++}$ and $\mathcal{A}_{+-,+-}$, are diagonal in the helicity basis. The third, $\mathcal{A}_{+-,-+}$, is off-diagonal (helicity flip: $\lambda = -\lambda'$).
Using the optical theorem we can write the distributions as:

$$f(x) = f_+(x) + f_-(x) \sim \mathrm{Im}(\mathcal{A}_{++,++} + \mathcal{A}_{+-,+-}) \tag{1.4.8a}$$

$$\Delta f(x) = f_+(x) - f_-(x) \sim \mathrm{Im}(\mathcal{A}_{++,++} - \mathcal{A}_{+-,+-}) \tag{1.4.8b}$$

$$\Delta_T f(x) = f_\uparrow(x) - f_\downarrow(x) \sim \mathrm{Im}\,\mathcal{A}_{+-,-+} \tag{1.4.8c}$$

In a transversity basis (with $\uparrow$ directed along y)

$$\begin{aligned} |\uparrow\rangle &= \tfrac{1}{\sqrt{2}}\left[|+\rangle + \mathrm{i}|-\rangle\right], \\ |\downarrow\rangle &= \tfrac{1}{\sqrt{2}}\left[|+\rangle - \mathrm{i}|-\rangle\right], \end{aligned} \tag{1.4.9}$$

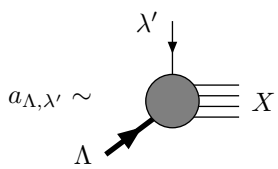
the transverse polarisation distributions $\Delta_T f$ is related to a diagonal amplitude

$$\Delta_T f(x) = f_\uparrow(x) - f_\downarrow(x) \sim \mathrm{Im}(\mathcal{A}_{\uparrow\uparrow,\uparrow\uparrow} - \mathcal{A}_{\uparrow\downarrow,\uparrow\downarrow})\,. \tag{1.4.10}$$

If transverse momenta of quarks are not neglected, the number of independent helicity amplitudes increases. The transverse-momentum dependent distributions are discussed in the next section.

1.5. *The Soffer inequality*

It is useful to introduce the quark-nucleon vertices $a_{\Lambda,\lambda'}$ (notice the sign of the elicity due to the incoming quark arrow):



Leading-twist distributions can be rewritten in the form

$$f(x) \sim \mathrm{Im}(\mathcal{A}_{++,++} + \mathcal{A}_{+-,+-}) \sim \sum_X (a^*_{++}a_{++} + a^*_{+-}a_{+-}) \quad (1.5.1a)$$

$$\Delta f(x) \sim \mathrm{Im}(\mathcal{A}_{++,++} - \mathcal{A}_{+-,+-}) \sim \sum_X (a^*_{++}a_{++} - a^*_{+-}a_{+-}) \quad (1.5.1b)$$

$$\Delta_T f(x) \sim \mathrm{Im}\,\mathcal{A}_{+-,-+} \sim \sum_X a^*_{--}a_{++} \quad (1.5.1c)$$

From the positivity constraint:

$$\sum_X |a_{++} \pm a_{--}|^2 \geq 0 \quad (1.5.2)$$

and using parity invariance, we obtain the inequality:

$$\sum_X a^*_{++}a_{++} \pm \sum_X a^*_{--}a_{++} \geq 0\,. \quad (1.5.3)$$

The latter can be written as:

$$f_+(x) \geq |\Delta_T f(x)| \quad (1.5.4)$$

which is equivalent to:

$$f(x) + \Delta f(x) \geq 2|\Delta_T f(x)| \quad (1.5.5)$$

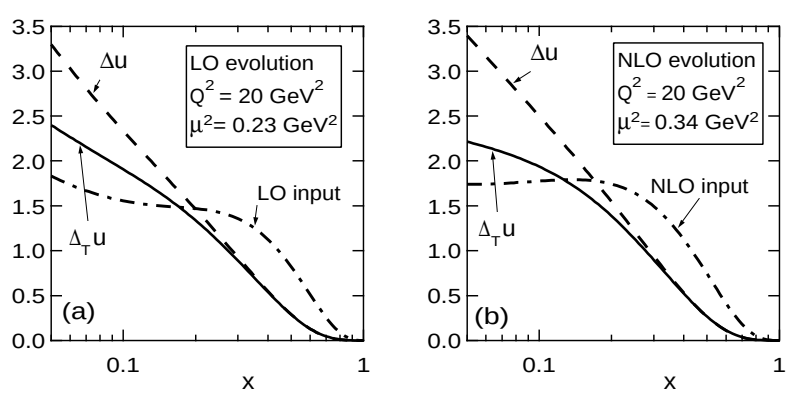
This is the so-called Soffer inequality [8].
Soffer inequality holds both for quarks and antiquarks and it is preserved by QCD evolution [9].

1.6. *QCD evolution*

The main features of the perturbative evolution of the transversity distrubutions are the following:

- The gluon field cannot contribute at LO in the case of spin-half hadrons as it would require helicity flip of two units in the corresponding hadron-parton amplitude. Thus, in the case of baryons the evolution is of purely non-singlet type.
- In contrast to the behaviour of both q and Δq, the anomalous dimensions governing $\Delta_T q$ do *not* vanish for $n = 1$ and hence there is no sum rule associated with the tensor charge
- Moreover $\Delta_T \gamma_{qq}(n) < \Delta\gamma_{qq}(n)$ for all n. This implies that for (hypothetically) identical starting distributions (*i.e.*, $\Delta_T q(x, Q_0^2) = \Delta q(x, Q_0^2)$), $\Delta_T q(x, Q^2)$ everywhere in x will fall more rapidly than $\Delta q(x, Q^2)$ with increasing Q^2 (see figure (from [10])).
- At LO the first moment of h_1 falls with increasing Q^2 as:

$$\begin{aligned}\delta q(Q^2) &= \left[\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right]^{-2\Delta_T\gamma_{qq}^{(0)}(1)/\beta_0} \delta q(Q_0^2) \\ &= \left[\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\right]^{-4/27} \delta q(Q_0^2) \qquad (1.6.1)\end{aligned}$$



2. Transverse-momentum dependent distributions

Transverse momentum is necessary in semi-inclusive DIS to study the $\boldsymbol{P}_{h\perp}$ distribution of the produced hadron.
The quark momentum is now given by

$$k^\mu \simeq x P^\mu + k_\perp^\mu \tag{2.0.1}$$

$k_\perp^\mu$ is zeroth order in P^+, and it is thus suppressed by one power of P^+ with respect to the longitudinal momentum.

2.1. *T-even distributions*

Taking into account the transverse momentum, the quark-quark correlation matrix then reads:

$$\begin{aligned}\Phi(k,P,S) = \frac{1}{2}\Big\{ & A_1 \not{P} + A_2\, \lambda_N\, \gamma^5 \not{P} + A_3 \not{P}\, \gamma_5 \not{S}_\perp \\ & + \frac{1}{M}\, \widetilde{A}_1\, \boldsymbol{k}_\perp\!\cdot\!\boldsymbol{S}_\perp\, \gamma^5 \not{P} + \widetilde{A}_2\, \frac{\lambda_N}{M}\, \not{P}\, \gamma^5\, \not{k}_\perp \\ & + \frac{1}{M^2}\, \widetilde{A}_3\, \boldsymbol{k}_\perp\!\cdot\!\boldsymbol{S}_\perp\, \not{P}\, \gamma^5\, \not{k}_\perp \Big\}\end{aligned} \tag{2.1.1}$$

If we do not integrate over $\boldsymbol{k}_\perp$, we obtain six $\boldsymbol{k}_\perp$-dependent distribution functions. Three of them are such that $f(x) = \int \mathrm{d}^2\boldsymbol{k}_\perp f(x, \boldsymbol{k}_\perp^2)$, *etc.* The other three are completely new

$$\Phi^{[\gamma^+]} = \mathcal{P}_{q/N}(x, \boldsymbol{k}_\perp) = f(x, \boldsymbol{k}_\perp^2) \tag{2.1.2a}$$

$$\begin{aligned}\Phi^{[\gamma^+\gamma^5]} &= \mathcal{P}_{q/N}(x, \boldsymbol{k}_\perp)\, \lambda(x, \boldsymbol{k}_\perp) \\ &= \lambda_N\, \Delta f(x, \boldsymbol{k}_\perp^2) + \frac{\boldsymbol{k}_\perp\!\cdot\!\boldsymbol{S}_\perp}{M}\, g_{1T}(x, \boldsymbol{k}_\perp^2)\end{aligned} \tag{2.1.2b}$$

$$\begin{aligned}\Phi^{[\mathrm{i}\sigma^{i+}\gamma^5]} &= \mathcal{P}_{q/N}(x, \boldsymbol{k}_\perp)\, s_\perp^i(x, \boldsymbol{k}_\perp) \\ &= S_\perp^i\, \Delta_T' f(x, \boldsymbol{k}_\perp^2) + \frac{\lambda_N}{M}\, k_\perp^i\, h_{1L}^\perp(x, \boldsymbol{k}_\perp^2) \\ &\quad - \frac{1}{M^2}\left(k_\perp^i k_\perp^j + \frac{1}{2}\, \boldsymbol{k}_\perp^2\, g_\perp^{ij}\right) S_{\perp j}\, h_{1T}^\perp(x, \boldsymbol{k}_\perp^2)\end{aligned} \tag{2.1.2c}$$

where $\mathcal{P}_{q/N}(x, \boldsymbol{k}_\perp)$ is the probability of finding a quark with longitudinal momentum fraction x and transverse momentum $\boldsymbol{k}_\perp$, and $\lambda(x, \boldsymbol{k}_\perp)$, $\boldsymbol{s}_\perp(x, \boldsymbol{k}_\perp)$ are the quark helicity and transverse spin densities, respectively. The three new (T-even) distributions are:
– $\boldsymbol{k}_\perp$-dependent transverse polarisation distributions of quarks in a transversely polarised nucleon

$$\mathcal{P}_{q\uparrow/N\uparrow}(x,\boldsymbol{k}_\perp) - \mathcal{P}_{q\downarrow/N\uparrow}(x,\boldsymbol{k}_\perp) = \cos(\phi_S - \phi_s)\,\Delta'_T f(x,\boldsymbol{k}_\perp^2) \quad (2.1.3)$$

$$+\,\frac{\boldsymbol{k}_\perp^2}{2M^2}\,\cos(2\phi_k - \phi_S - \phi_s)\,h_{1T}^\perp(x,\boldsymbol{k}_\perp^2) \quad (2.1.4)$$

– longitudinal polarisation distribution of quarks in a transversely polarised nucleon

$$\mathcal{P}_{q+/N\uparrow}(x,\boldsymbol{k}_\perp) - \mathcal{P}_{q-/N\uparrow}(x,\boldsymbol{k}_\perp) = \frac{|\boldsymbol{k}_\perp|}{M}\,\cos(\phi_S - \phi_k)\,g_{1T}(x,\boldsymbol{k}_\perp^2) \quad (2.1.5)$$

– transversely polarised quark distribution in a longitudinally polarised nucleon

$$\mathcal{P}_{q\uparrow/N+}(x,\boldsymbol{k}_\perp) - \mathcal{P}_{q\downarrow/N+}(x,\boldsymbol{k}_\perp) = \frac{|\boldsymbol{k}_\perp|}{M}\,\cos(\phi_k - \phi_s)\,h_{1L}^\perp(x,\boldsymbol{k}_\perp^2) \quad (2.1.6)$$

2.2. *T-odd distributions*

Relaxing the time-reversal invariance condition two additional terms in the vector and tensor components of Φ arise, which give rise to two $\boldsymbol{k}_\perp$-dependent T-odd distribution functions

$$\Phi^{[\gamma^+]} = \cdots - \frac{\varepsilon_\perp^{ij} k_{\perp i} S_{\perp j}}{M}\,f_{1T}^\perp(x,\boldsymbol{k}_\perp^2) \quad (2.2.1a)$$

$$\Phi^{[\mathrm{i}\sigma^{i+}\gamma_5]} = \cdots - \frac{\varepsilon_\perp^{ij} k_{\perp j}}{M}\,h_1^\perp(x,\boldsymbol{k}_\perp^2) \quad (2.2.1b)$$

the physical interpretation of the two T-odd distributions is:
– number density of unpolarised quarks in a transversely polarised nucleon [11, 12]

$$\mathcal{P}_{q/N\uparrow}(x,\boldsymbol{k}_\perp) - \mathcal{P}_{q/N\downarrow}(x,\boldsymbol{k}_\perp) = \mathcal{P}_{q/N\uparrow}(x,\boldsymbol{k}_\perp) - \mathcal{P}_{q/N\uparrow}(x,-\boldsymbol{k}_\perp)$$

$$= -2\frac{|\boldsymbol{k}_\perp|}{M}\,\sin(\phi_k - \phi_S)\,f_{1T}^\perp(x,\boldsymbol{k}_\perp^2) \quad (2.2.2a)$$

– quark transverse polarisation in an unpolarised hadron [13]

$$\mathcal{P}_{q\uparrow/N}(x,\boldsymbol{k}_\perp) - \mathcal{P}_{q\downarrow/N}(x,\boldsymbol{k}_\perp) = -\frac{|\boldsymbol{k}_\perp|}{M}\,\sin(\phi_k - \phi_s)\,h_1^\perp(x,\boldsymbol{k}_\perp^2) \quad (2.2.2b)$$

2.3. *Why the so-called T-odd distributions can be non-vanishing?*

We come back to eqs.(1.1.1–1.1.3), to clarify the physical meaning of the link operator. It represents the final state interaction in the eikonal approximation between the struck quark and the gluon field in the target spectator [4]. The link $\mathcal{L}(y, \infty)$ of eq.(1.1.2) integrates from the point y to future infinity. The crucial observation is that, under time-reversal the future-pointing Wilson lines are replaced by past-pointing ones. The latter are the appropriate choice for factorization in Drell-Yan processes. This is the reason why it was *naive* to assume eq.(1.1.6) as the appropriate one to describe the time-reversal transformation of the correlation function (and similarly for the distribution functions).
The previous argument by Collins is based on QCD and it shows that "T-odd" distribution functions are in principle non-vanishing, since they actually are *not* T-odd! In practical calculations of distribution functions, effective lagrangians are used instead of QCD, since at a low scale effective degrees of freedom take the place of the QCD ones. For instance chiral lagrangians can be used, were the effective degrees of freedom are quarks and chiral fields instead of quarks and gluons. How is it then possible to have non-vanishing T-odd distribution functions in those effective models? A possible answer is that in chiral models the representation of the time-reversal operator is more complicated than the usual one, and it mixes the components of the up-down doublet [14].
The problem of the physical origin of T-odd distribution functions in QCD has also been discussed in other very recent papers, and I suggest to follow the rapidly expanding literature on it [15].

3. Conclusions

The transverse polarisation of hadrons opens many interesting questions both from the theoretical and the experimental viewpoint.
For theoreticians, questions related to single-spin asymmetries and in particular to T-odd distribution (and fragmentation) functions are of extreme interest, both for finding which constraints can be obtained directly from QCD and also for the practical (but challenging) purpose of computing them within an effective model.
For experimentalists, the measure of the leading-twist transversity distribution $\Delta_T f(x)$ is probably the most important task, together with the study of semi-inclusive DIS reactions, which will eventually allow to clarify the

questions related to T-odd distribution and fragmentation functions.

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