EFFECTIVE NUCLEON POLARIZATION IN POLARIZED TARGETS

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The nucleon spin structure functions have been extracted from measurements of asymmetries in deep inelastic scattering of polarized leptons on polarized nuclei in several experiments at CERN, SLAC and DESY. Except for hydrogen, the polarized nuclei present in practical targets: ^{2}H, ^{3}He, ^{14}N, ^{15}N, ^{6}Li and ^{7}Li, are systems of bound nucleons, some of which can attain significant degrees of alignment. All the aligned nucleons contribute to the asymmetries. The contributions of each nuclear species to the asymmetry have to be carefully determined, before a reliable value for the net nucleon asymmetry is obtained. For this purpose, the spin component of the nuclear angular momentum for every nuclear state and the probability of each state have to be known with sufficient accuracy. In this talk I discuss the basic corrections used to estimate the contributions of the different nuclei, using as example deuterium and ^{6}Li which are present in the Li^{2}H polarized target used during SLAC Experiment 155 to study the deuteron spin structure.

1. Introduction

Polarized deep inelastic scattering (DIS) experiments[1,2,3,4,5,6,7] study the properties of the nucleon using polarized nuclear targets. One needs a way of relating the polarization of the nuclei in the target to the effective polarization of the nucleons in the polarized nuclei.

Some useful pieces of information to this effect are the facts that the nuclear polarization depends on the magnitude of the nuclear magnetic moment μ, and that the nucleon polarization depends on the angular momentum composition of the magnetic moment. Based on this information, a simple approach to the effective nucleon polarization can be formulated as follows: find the fractional spin component of the magnetic moment for each nuclear level of interest, and take the nucleon polarization as the sum of these spin contributions to the magnetic moment. This approach is valid in the kinematic range for which little or no kinematic dependence of

the nucleon polarization is expected. In terms of the usual DIS kinematic variables four-momentum transfer squared $-Q^2$ and Bjorken scaling variable x, this means $Q^2 \geq 1$ (GeV/c)^2 and $0.015 \leq x \leq 0.75$, which is the experimentally measured range at SLAC[7].

Two methods can be followed to solve for the spin component of μ: model independent and model dependent. The first one relies on simple symmetries, such as isospin, and can be applied to low Z nuclei[8,9]. In the second one, a weighted sum over angular momentum states is formed, using weights derived from a variety of models. The model dependent method can be applied to a wide variety of nuclei.

2. Model independent method

2.1. *Isosinglets*

For the case of isosinglet, or self-conjugate, nuclei (^{2}H, ^{6}Li and ^{14}N) the magnetic moment and the angular momentum can be written in terms of the spin $S_z^{(p,n)}$ and orbital $L_z^{(p)}$ components as[9]

$$\mu = g_s^p \langle S_z^{(p)} \rangle + g_s^n \langle S_z^{(n)} \rangle + g_l^p \langle L_z^{(p)} \rangle \tag{1}$$

$$I = \langle S_z^{(p)} \rangle + \langle S_z^{(n)} \rangle + \langle L_z^{(p)} \rangle + \langle L_z^{(n)} \rangle \tag{2}$$

where $g_s^{p(n)} = 2\mu_{p(n)} = 2(2.793, -1.913)$ are free nucleon g-factors. Solving for the spin component and using isospin symmetry one gets

$$\langle S_z^{(p)} \rangle = \langle S_z^{(n)} \rangle = \frac{(\mu - \frac{I}{2})}{0.76}. \tag{3}$$

Substituting μ in eq. 3 one gets the results shown in Table 1

Table 1. Spin 1 nuclei.

Nuclide	Magnetic moment [μ_N]	Spin component	Orbital component ($\times 1/2$)
^{2}H	0.857	0.94	0.03
^{6}Li	0.822	0.847	0.077
^{14}N	0.404	-0.26	0.63

These results will be compared with those of the model dependent method at the end.

2.2. *Other mirror nuclei*

Isospin symmetry can also be applied to the mirror doublets (^{3}H – ^{3}He, ^{7}Li – ^{7}Be, ^{15}N – ^{15}O). In this case, one solves the sum and the difference of the magnetic moments of the doublet for the spin component

$$\begin{aligned}\mu(t_3=\frac{1}{2})+\mu(t_3=-\frac{1}{2}) &= \langle\sum_i l_z^{(i)}\rangle+(\mu_p+\mu_n)\langle\sum_i \sigma_z^{(i)}\rangle \\ \mu(t_3=\frac{1}{2})-\mu(t_3=-\frac{1}{2}) &= \langle\sum_i \tau_3 l_z^{(i)}\rangle+(\mu_p-\mu_n)\langle\sum_i \tau_3\sigma_z^{(i)}\rangle. \end{aligned} \tag{4}$$

The spin term in the difference is related to the Gamow-Teller β decay matrix element[10,11,12]

$$\left(\frac{g_A}{g_V}\right)^2\left|\langle\sum_i \tau_3\sigma_z^{(i)}\rangle\right|^2=\frac{ft}{6170}-1. \tag{5}$$

For the pair ^{7}Li – ^{7}Be, the magnetic moment of the radioactive ^{7}Be (half-life: 53.3 days) disappointingly has not been measured. Results for the other two pairs are shown at the end.

3. Model dependent method

In the model dependent method, the nuclear ground state wave function is expanded in terms of angular momentum eigenstates[13,8]

$$\psi=\sum_{L,S} c_{LS}\psi_{LS} \tag{6}$$

where the coefficients $|c_{LS}|^2=P_L$ represent normalized probabilities $P_L=N_L/N$, $N=\sum N_L$, of occurrence of the L,S states. The usual notation for the states is $\psi_{LS}=\text{CG}\,|m_L,m_S;M\rangle$ where CG represents Clebsch-Gordan coefficients.

The nucleon polarization is the fraction of nucleons aligned with the nuclear spin. The polarization for each level is $p_{p,n}^{(L)}=(N_L^{\uparrow\uparrow}-N_L^{\uparrow\downarrow})/N_L$, where $N_L^{\uparrow\uparrow}(N_L^{\uparrow\downarrow})$ are the number of nucleons in level L with their spins parallel (anti parallel) to the spin of the nucleus, and the effective polarization is $p_{p,n}=\sum P_L\ p_{p,n}^{(L)}$. The examples below illustrate how the method works.

3.1. *Deuterium*

The deuteron is in the $I^\pi=1^+$ spin triplet state (nucleons with parallel spins). Only even components of orbital angular momentum are allowed.

The total angular momentum is formed by L–S coupling $L = \ell_p + \ell_n$ and $S = s_p + s_n$. In the D state $I = L + S = 2 \otimes 1$.

With this notation the S state and the $D, M = +1$ substate can be written as

$$\psi_{0,1} = c_{0,1}|0,1;1\rangle$$
$$\psi_{2,1} = c_{2,1}\Big(\sqrt{\tfrac{3}{5}}|2,-1;1\rangle + \sqrt{\tfrac{3}{10}}|1,0;1\rangle + \sqrt{\tfrac{1}{10}}|0,1;1\rangle\Big)\,, \tag{7}$$

with a similar expression for the $M = -1$ substate. By inspection of the relative signs of the m_S and M quantum numbers one sees that in the D state the nucleons are anti parallel to the deuteron spin 3/5 of the time, and parallel 1/10. The nucleons in the $m_S = 0$ substates have zero net spin. The nucleon polarization in the D level is then

$$p_{p,n}^{(D)} = \frac{N_D^{\uparrow\uparrow}}{N_D} - \frac{N_D^{\uparrow\downarrow}}{N_D} = \frac{1}{10} - \frac{3}{5} = -\frac{1}{2}\,, \tag{8}$$

and the effective nucleon polarization in deuterium is

$$p_{p,n} = P_S\; p_{p,n}^{(S)} + P_D\; p_{p,n}^{(D)} = \frac{1}{N}(N_S - \frac{1}{2}N_D)$$
$$p_{p,n} = \qquad 1 - 1.5P_D \qquad = 0.926 \pm 0.016\,, \tag{9}$$

with $N = N_S + N_D$. The D state probability $P_D = 0.0490 \pm 0.0104$ is an average of five models[14]. This constant value for the effective proton or neutron polarization is good for most of the range in x.

3.2. *Lithium*

^{6}Li is another spin 1 nucleus $I^\pi = 1^+$, but since it is a system with more than two bodies additional angular momentum states are allowed. In a 3-body configuration consisting of an α particle core plus a proton and a neutron, S, P and D levels are allowed. The S and D states for the proton and neutron outside the core are as in ^{2}H: ^{6}Li can be effectively treated as a pair of $\alpha + d$ clusters. The P state has two substates, singlet and triplet. In the singlet state, the nucleons have zero net spin. In the triplet state, $M = +1$ substate one has

$$\psi_{1,1} = c_{1,1}\Big(\sqrt{\frac{1}{2}}|1,0;1\rangle - \sqrt{\frac{1}{2}}|0,1;1\rangle\Big)\,. \tag{10}$$

Only one half of the nucleons in this state are aligned with the Li spin, so $p_{p,n}^{P(M=\pm1)} = 1/2$. The nucleons in substates with $m_S = 0$ have their spins equally aligned in opposite directions.

The effective nucleon polarization in ^{6}Li is

$$\begin{aligned} p_{p,n} &= P_S\ p_{p,n}^{(S)} + P_{P0}\ p_{p,n}^{(P0)} + P_{P1}\ p_{p,n}^{(P1)} + P_D\ p_{p,n}^{(D)} \\ &= \frac{1}{N}(N_S + 0 + \frac{1}{2}N_{P1} - \frac{1}{2}N_D) \\ &= 1 - P_{P0} - 0.5P_{P1} - 1.5P_D\,. \end{aligned} \tag{11}$$

The P_L probabilities have been calculated in ref.[15] by solving three-body Fadeev equations for a variety of NN and αN potentials. Taking the average of those results for eight models that predict $\mu(^6Li)$ to better than 2%, the probabilities are[14] $P_{P0} = 0.0238$, $P_{P1} = 2.9 \times 10^{-3}$ and $P_D = 0.0725$, which result in a nucleon polarization in ^{6}Li of $(0.866\pm0.012)/3$. The uncertainty quoted is the standard deviation of the results of the individual models.

A Green's functions Monte Carlo calculation[16] obtained the internal nucleon momentum dependence of the polarization for ^{3}He, ^{6}Li and ^{7}Li. It predicts integrated nucleon spin densities of 1.93 for spin up and 1.07 for spin down in ^{6}Li with its nuclear spin up, or a net nucleon polarization of 0.86/3. The simplified estimate of ref.[17] for the same quantity is 0.82/3.

Shell and cluster models also give reasonable numbers for ^{7}Li and for the N isotopes. In these cases, a kinematics independent nucleon polarization is obtained. Additional effects must be included to describe the nucleon polarization at small x. A recent work[18] includes the effects of shadowing and isobar configurations in ^{3}He.

4. Summary

Results for several nuclear species, including the examples discussed above are summarized in Table 2 below.

* $p(n)$ in ^{3}H (^{3}He)

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Table 2. Nucleon Polarization.

Nuclide	Model Independent		Model Dependent					
	Ref.[14]	Ref.[10]	Ref.[14]	Ref.[18]	Ref.[19]	Ref.[15]	Ref.[16]	Ref.[17]
^{2}H	0.94		0.926					
^{3}H-^{3}He*	0.93	0.96	0.86	0.879	0.865			
^{6}Li	0.85		0.866			0.866	0.86	0.82
^{7}Li			0.57				0.59	
^{14}N	-0.26		-0.33					
^{15}N	-0.22	-0.24	-0.33					

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