E93-026 Technical Note:

Beam Current Asymmetry

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Introduction: The goal is to find out how long one has to measure to achieve a certain accuracy in a measurement of beam current asymmetry. For this purpose asymmetries are calculated for single helicity pairs and width and shape of the distribution is studied.

Asymmetries: Beam current is calculated from scaler values and is given as charge divided by clock. It turns out that the clock asymmetries are more than an order of magnitude smaller than the charge asymmetries. Thus charge asymmetries are calculated for simplicity (they are already available from an earlier analysis).
The charge asymmetries A_i are calculated for each helicity pair for run 21937, which has an average beam current of 85 nA. Each helicity pair corresponds to a time interval of 2 seconds.

Statistics: Here some elementary statistics:
Mean: $\overline{A} = -7.88 \cdot 10^{-4}$
Standard deviation: $\sigma = 3.25 \cdot 10^{-3}$

Fit: The A_i's are histogrammed vs. $(A_i - \overline{A})/\sigma$ and a Gauss curve is fitted to the distribution leaving the width ξ of the function variable (see figure 1).
The result of the fit:
Width: $\xi = 0.96$
$\chi^2_{red} = 0.95$

Result: The distribution of charge asymmetries is well represented by a normal probability distribution and the absolute error of the measurement is given by the standard deviation σ. Adding up charge values for N helicity pairs should lead then to an improved accuracy by a factor $1/\sqrt{N}$. To verify this the above procedure was repeated for N=4, 9, 16 and 25. The results are summarized in the table below and show that the agreement with expectations is fairly good.

	Standard deviation σ		Fit	
	Expected $\sigma(N=1)/\sqrt{N}$	Calculated	ξ	χ^2_{red}
N=1		$3.25 \cdot 10^{-3}$	0.96	0.95
N=4	$1.63 \cdot 10^{-3}$	$1.74 \cdot 10^{-3}$	1.02	0.94
N=9	$1.08 \cdot 10^{-3}$	$1.27 \cdot 10^{-3}$	1.07	0.79
N=16	$0.81 \cdot 10^{-3}$	$0.91 \cdot 10^{-3}$	1.02	0.53
N=25	$0.65 \cdot 10^{-3}$	$0.66 \cdot 10^{-3}$	1.17	0.54

Example: To reach an absolute error better than 10^{-3} in a beam current asymmetry measurement one has to measure roughly half a minute (corresponds to N=15). This is true for a beam current of 85 nA.

Large N: To test for larger N approximately 150 runs were added together and the above exercise was repeated again for N up to 8100. The table below lists the results. Fig. 2 bottom shows the calculated and the fitted standard deviation vs. $\sqrt{N}$ as well as the expectation curves. For small N the fitted and calculated standard deviation disagree (see fig. 2 top for N=1), which is due to more pronounced tails in the distribution. This is not surprising considering the long time period over which runs have been added up here. It seems to cancel out with averaging and the distribution becomes more gaussian again. The agreement with expectation is still good and supports the findings for the single run above.

	Standard deviation σ		Fit	
	Expected $\sigma(N=1)/\sqrt{N}$	Calculated	ξ	χ^2_{red}
N=1		$6.32 \cdot 10^{-3}$	0.60	53.1
N=25	$1.26 \cdot 10^{-3}$	$1.26 \cdot 10^{-3}$	0.65	8.9
N=100	$6.32 \cdot 10^{-4}$	$6.25 \cdot 10^{-4}$	0.78	4.7
N=400	$3.16 \cdot 10^{-4}$	$2.98 \cdot 10^{-4}$	0.92	1.0
N=900	$2.11 \cdot 10^{-4}$	$1.86 \cdot 10^{-4}$	1.03	0.4
N=1600	$1.58 \cdot 10^{-4}$	$1.45 \cdot 10^{-4}$	0.76	1.4
N=2500	$1.26 \cdot 10^{-4}$	$1.18 \cdot 10^{-4}$	0.69	2.4
N=3600	$1.05 \cdot 10^{-4}$	$1.14 \cdot 10^{-4}$	1.27	0.4
N=4900	$9.03 \cdot 10^{-5}$	$8.94 \cdot 10^{-5}$	1.00	0.7
N=6400	$7.90 \cdot 10^{-3}$	$7.37 \cdot 10^{-5}$	1.22	0.6
N=8100	$7.02 \cdot 10^{-5}$	$7.01 \cdot 10^{-5}$	1.65	0.7

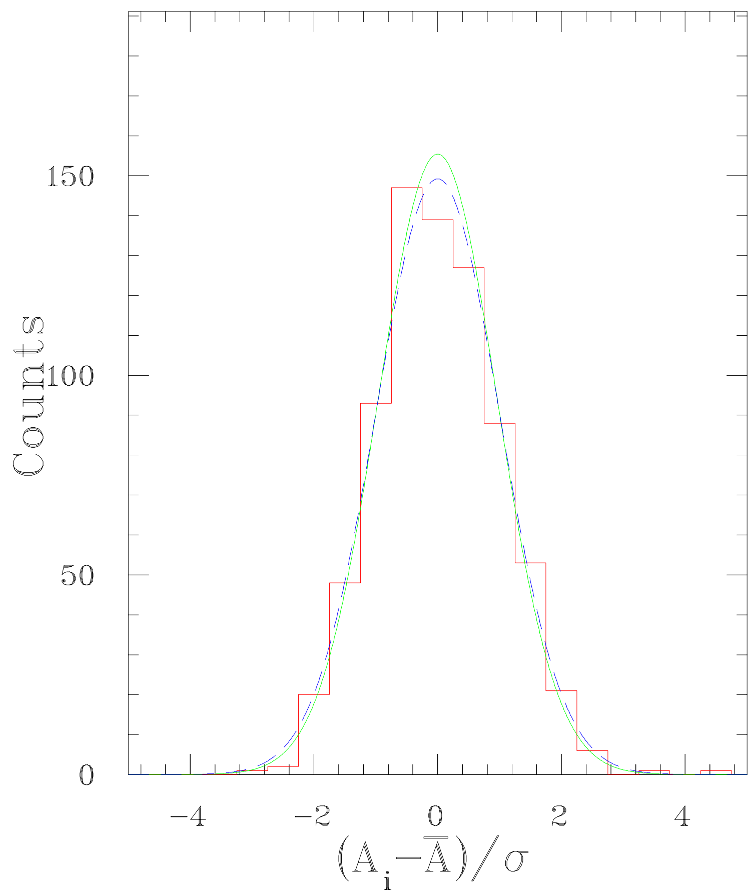

Figure 1: *Histogrammed deviation of charge asymmetries A_i from the mean $\overline{A}$ in units of the standard deviation σ. The green curve is the fit of a gaussian function to the distribution leaving the width variable. The blue dashed curve is a gaussian function with width 1.*

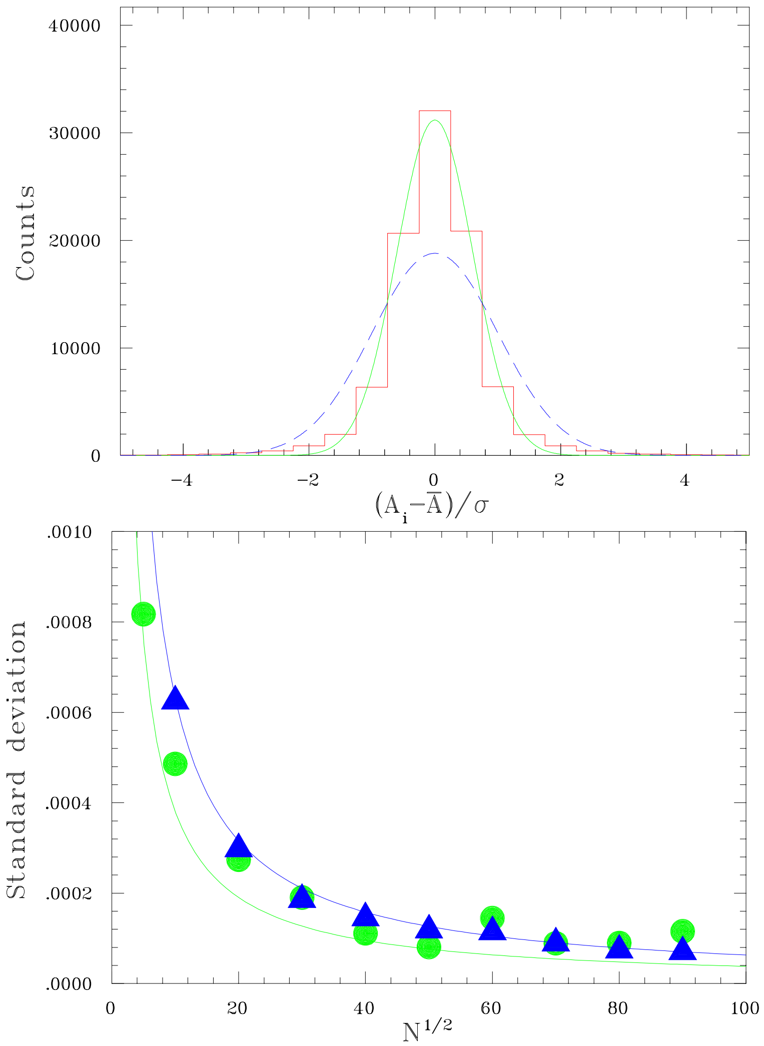

Figure 2: *Top: Same as fig. 2 but now for a large sample of helicity packets from approximately 150 runs. The deviation between calculated and fitted standard deviation is due to the more pronounced tails of the distribution. Bottom: Standard deviation vs. number of helicity pairs. The blue points are calculated σ, while the green points are the fitted width of the distribution ($\xi \cdot \sigma$). The lines represent the* $1/\sqrt{N}$ *expectation curves normalized to N=1.*