G_E^n Technical Note
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E93-026 Asymmetry Calculation

We develop the general formalism for the calculation of asymmetries without making any assumptions on the strength of the different contributions. This is based on a previous TN by Oscar A. Rondon[1].

1 Definitions

Polarized cross section:

$$\sigma(h,P,T)=\sigma_0\left(1+hA_e+PA_d^V+TA_d^T+h\left(PA_{ed}^V+TA_{ed}^T\right)\right) \tag{1}$$

with beam helicity h, vector target polarization P, Tensor target polarization T which is a function of $P: T=2-\sqrt{4-3P^2}$

Based on measured counts N^{hP} for different combination of h and P we can define different asymmetries:

Beam-Target Asymmetry:

$$A_{BT}=\frac{N^{++}-N^{-+}+N^{--}-N^{+-}}{N^{++}+N^{-+}+N^{--}+N^{+-}} \tag{2}$$

Beam asymmetry for positive target polarization:

$$A_{B+}=\frac{N^{++}-N^{-+}}{N^{++}+N^{-+}} \tag{3}$$

Beam asymmetry for negative target polarization:

$$A_{B-}=\frac{N^{--}-N^{+-}}{N^{--}+N^{+-}} \tag{4}$$

[1] Oscar A. Rondon: "A_{ed}^V for all combinations of beam and target polarizations" G_E^n TN 2000-10, Oct. 2000

2 Summation and Normalization

Measured counts for possible combinations of h and P are given as:

$$N^{hP} \propto Q^{hP} \cdot L^{hP} \cdot \left(\sum_j n_j \sigma_j \right)$$

Q: Charge, L: Life-time, n_j: number of target nuclei of type j involved, σ_j: cross sections. All other factors like detector acceptance are considered as constant and not dependent on h or P.

To keep the expressions for the measured asymmetries as simple as possible it is most favourable to divide the data into sets for which the polarizations can be considered as constant and to normalize each set individually. In the following we call a data set of constant polarization a run.

Assuming that we have m runs with N_i^{hP} (i=1..m) counts for the different combinations of h and P the normalized sum becomes

$$N^{hP} = \frac{1}{m} \sum_{i=1}^{m} \frac{N_i^{hP}}{Q_i^{hP} L_i^{hP}} \tag{5}$$

3 Calculation of Asymmetries

Starting with the simple, idealized case we generalize step by step.

3.1 Constant polarizations, only deuterium

Assuming that polarized deuteron is the only nucleus involved in the scattering process makes the normalized counts proportional to the deuterium cross section σ_D:

$$N^{hP} \propto \sigma_D(h, P, T)$$

Resulting beam-target asymmetry (eq. 2) for constant P and h:

$$A_{BT} = \frac{hPA_{ed}^V}{1 + TA_d^T}$$

The appearance of A_d^T in the denominator reflects the fact that T does not change sign with P.

Beam asymmetry (eq. 3 and 4):

$$A_{B\pm} = \frac{hPA_{ed}^V \pm hA_e \pm hTA_{ed}^T}{1 + TA_d^T \pm PA_d^V}$$

Average of Beam asymmetries:

$$\frac{1}{2}\left(A_{B+}+A_{B-}\right)=\frac{\left(1+TA_{d}^{T}\right)hPA_{ed}^{V}-\left(hA_{e}+hTA_{ed}^{T}\right)PA_{d}^{V}}{\left(1+TA_{d}^{T}\right)^{2}-\left(PA_{d}^{V}\right)^{2}}$$

This expression becomes identical to A_{BT} if one neglects the higher order (square) asymmetry terms .

3.2 Variable polarizations

Polarizations (h, P) not constant but changing over time. No other nuclei than deuterium involved. Using eq. 5 for summation and normalization leads makes the normalized counts proportional to the deuteron cross section σ_D only:

$$N^{hP}\propto\sigma_{D}(h,P,T)$$

Beam-target asymmetry (eq. 2):

$$A_{BT}=\frac{\langle hP\rangle A_{ed}^{V}+\langle h\rangle A_{e}+\langle hT\rangle A_{ed}^{T}}{1+\langle P\rangle A_{d}^{V}+\langle T\rangle A_{ed}^{T}}$$

1..k (k+1..m) runs with positive (negative) target polarization and assuming that positive and negative beam helicity are the same. For the averaged polarizations we get:

$$\langle hP\rangle = \frac{1}{2}\left(\frac{1}{k}\sum_{i=1}^{k}h_iP_i-\frac{1}{m-k}\sum_{i=k+1}^{m}h_iP_i\right) \quad (6)$$

$$\langle hT\rangle = \frac{1}{2}\left(\frac{1}{k}\sum_{i=1}^{k}h_iT_i-\frac{1}{m-k}\sum_{i=k+1}^{m}h_iT_i\right) \quad (7)$$

$$\langle P\rangle = \frac{1}{2}\left(\frac{1}{k}\sum_{i=1}^{k}P_i+\frac{1}{m-k}\sum_{i=k+1}^{m}P_i\right) \quad (8)$$

$$\langle T\rangle = \frac{1}{2}\left(\frac{1}{k}\sum_{i=1}^{k}T_i+\frac{1}{m-k}\sum_{i=k+1}^{m}T_i\right) \quad (9)$$

$$\langle h\rangle = \frac{1}{2}\left(\frac{1}{k}\sum_{i=1}^{k}h_i-\frac{1}{m-k}\sum_{i=k+1}^{m}h_i\right) \quad (10)$$

3.3 Another unpolarized nucleus

Assuming a target consisting of 2 types of nuclei: n_D polarized deuteron nuclei with cross section σ_D and n_N unpolarized nuclei with cross section σ_N. Using eq. 5 for summation and normalization leads to:

$$N^{hP} \propto n_D \sigma_D(h,P,T) + n_N \sigma_N(h)$$

Beam-target asymmetry (eq. 2) with average polarizations according to equations 6-10:

$$A_{BT} = \frac{n_D \sigma_0^{(D)} \left[\langle hP\rangle A_{ed}^V + \langle h\rangle A_e + \langle hT\rangle A_{ed}^T\right] + n_N \sigma_0^{(N)} \left[\langle h\rangle A_e^{(N)}\right]}{n_N \sigma_0^{(N)} + n_D \sigma_0^{(D)} \left[1 + \langle P\rangle A_d^V + \langle T\rangle A_{ed}^T\right]}$$

Assuming that $\langle h\rangle A_e^{(N)} = 0$ the asymmetry can be expressed in another form:

$$A_{BT} = f \frac{\langle hP\rangle A_{ed}^V + \langle h\rangle A_e + \langle hT\rangle A_{ed}^T}{1 + \langle P\rangle A_d^V + \langle T\rangle A_{ed}^T} \tag{11}$$

with the dilution factor f

$$f = \frac{n_D \sigma_0^{(D)}}{\frac{n_N \sigma_0^{(N)}}{1+\langle P\rangle A_d^V + \langle T\rangle A_d^T} + n_D \sigma_0^{(D)}} \tag{12}$$

The concept of dilution factor according to equations 11 and 12 is only applicable if the terms of the "diluting" nucleus are limited to the denominator, otherwise the factorization does not work.

3.4 Other polarized nuclei

Assuming a target consisting of L types of polarized, each of count $n_{(j)}$ and with cross section $\sigma_{(j)}$ of the form according to eq. 1. Using eq. 5 for summation and normalization leads to:

$$N^{hP} \propto \sum_{j=1}^{L} n_{(j)} \sigma_{(j)}(h,P,T)$$

Beam-target asymmetry (eq. 2) with average polarizations according to equations 6-10:

$$A_{BT} = \frac{\sum_{j=1}^{L} n_{(j)} \sigma_0^{(j)} \left[\langle hP\rangle A_{ed}^{V(j)} + \langle h\rangle A_e^{(j)} + \langle hT\rangle A_{ed}^{T(j)}\right]}{\sum_{j=1}^{L} n_{(j)} \sigma_0^{(j)} \left[1 + \langle P\rangle A_d^{V(j)} + \langle T\rangle A_{ed}^{T(j)}\right]}$$

3.5 Variable packing fraction

A change of target density at constant target volume corresponds to a change in $n_{(j)}$ which can in general be different for each run i: $n_{(j)i}$.

In this case the averaging of the polarizations gets affected too and the formalism looks like the following:

Averaged $n_{(j)i}$:

$$\langle n_{(j)} \rangle = \frac{1}{2}\left(\frac{1}{k}\sum_{i=1}^{k} n_{(j)i} + \frac{1}{n-k}\sum_{i=k+1}^{n} n_{(j)i}\right) \tag{13}$$

Averaged polarizations:

$$\langle hP \rangle_{(j)} = \frac{1}{2\langle n_{(j)} \rangle}\left(\frac{1}{k}\sum_{i=1}^{k} n_{(j)i}h_i P_i - \frac{1}{m-k}\sum_{i=k+1}^{m} n_{(j)i}h_i P_i\right) \tag{14}$$

$$\langle hT \rangle_{(j)} = \frac{1}{2\langle n_{(j)} \rangle}\left(\frac{1}{k}\sum_{i=1}^{k} n_{(j)i}h_i T_i - \frac{1}{m-k}\sum_{i=k+1}^{m} n_{(j)i}h_i T_i\right) \tag{15}$$

$$\langle P \rangle_{(j)} = \frac{1}{2\langle n_{(j)} \rangle}\left(\frac{1}{k}\sum_{i=1}^{k} n_{(j)i} P_i + \frac{1}{m-k}\sum_{i=k+1}^{m} n_{(j)i} P_i\right) \tag{16}$$

$$\langle T \rangle_{(j)} = \frac{1}{2\langle n_{(j)} \rangle}\left(\frac{1}{k}\sum_{i=1}^{k} n_{(j)i} T_i + \frac{1}{m-k}\sum_{i=k+1}^{m} n_{(j)i} T_i\right) \tag{17}$$

$$\langle h \rangle_{(j)} = \frac{1}{2\langle n_{(j)} \rangle}\left(\frac{1}{k}\sum_{i=1}^{k} n_{(j)i} h_i - \frac{1}{m-k}\sum_{i=k+1}^{m} n_{(j)i} h_i\right) \tag{18}$$

Beam-target asymmetry (eq. 2) with averaged $n_{(j)i}$ according to eq. 13 and averaged polarizations according to eq. 14 to 18:

$$A_{BT} = \frac{\sum_{j=1}^{L} \langle n_{(j)} \rangle \sigma_0^{(j)} \left[\langle hP \rangle_{(j)} A_{ed}^{V(j)} + \langle h \rangle_{(j)} A_e^{(j)} + \langle hT \rangle_{(j)} A_{ed}^{T(j)}\right]}{\sum_{j=1}^{L} \langle n_{(j)} \rangle \sigma_0^{(j)} \left[1 + \langle P \rangle_{(j)} A_d^{V(j)} + \langle T \rangle_{(j)} A_{ed}^{T(j)}\right]}$$