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Asymmetries in $e-nucleon$ Inclusive Scattering for All Combinations of Beam and Target Polarizations, Including Electroweak Interference.

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Contents

1 Longitudinal asymmetry

The complete expression for deep inelastic inclusive lepton-nucleon scattering cross sections, for neutral currents nc, is given in [1] as a function of three unpolarized structure functions ($F_1(x,Q^2), F_2(x,Q^2)$ and $F_3(x,Q^2)$) and five polarized ones ($g_k(x,Q^2), k=1,..,5$). For longitudinal beam polarization $\lambda = \pm P_b$ and longitudinal target polarization P_t the cross section in a convenient compact notation is

$$\frac{d^2\sigma_{nc}^{lN}}{dxdy}(x,Q^2,\lambda,P_t) = 8\pi MxyE\frac{\alpha^2}{Q^4}\sum_{i=\gamma,\gamma Z,Z}\eta^i C^i$$

$$\Big[k_1F_1^i + \frac{k_2}{x}F_2^i - \lambda k_3 F_3^i$$

$$-\lambda P_t(k_4 g_1^i - 2k_5 g_2^i) + P_t(\frac{k_2}{x}g_3^i - \frac{k_6}{x}g_4^i + k_7 g_5^i)\Big]. \quad (1)$$

The index i represents the type of structure function (electromagnetic, interference or weak). The coefficients $\eta^\gamma = 1$, $\eta^{\gamma Z} = (GM_z^2)/(\sqrt{8}\pi\alpha)(Q^2/(Q^2+M_Z^2)$ and $\eta^Z = (\eta^{\gamma Z})^2$ represent the relative couplings, and the $C^\gamma = 1$, $C^{\gamma Z} = g_V - \lambda g_A$, and $C^Z = (C^{\gamma Z})^2$ are related to the vector $g_V = -1/2 + 2\sin^2\theta_W$ and axial vector $g_A = -1/2$ components. The symbols

x, y, M, Q^2 and E have their usual DIS kinematics meanings, and the kinematics coefficients k_m are

$$
\begin{aligned}
k_1 &= y \\
k_2 &= \frac{1}{y}\Big(1 - y - \frac{xyM}{2E}\Big) \\
k_3 &= 1 - \frac{y}{2} \\
k_4 &= 2 - y - \frac{xyM}{E} \\
k_5 &= \frac{xM}{E} \\
k_6 &= \frac{1}{y}\Big(1 - y - \frac{xyM}{2E}\Big)\Big(1 + \frac{xM}{E}\Big) \\
k_7 &= y\Big(1 + \frac{xM}{E}\Big).
\end{aligned}
\tag{2}
$$

The numerical values of the η^i coefficients determine which contributions are important at a given kinematics. For the SLAC spin structure experiments, at an average $Q^2 \sim 5$ GeV^2 and maximum $Q^2\sim$ 30 GeV^2, the weak contributions $\eta^Z \leq \; \sim 2 \times 10^{-5}$ can be entirely neglected. Even the electroweak contributions are very small $\eta^{\gamma Z}(Q^2 = 30 \text{ GeV}^2) = 0.0048$, or 0.5% of the electromagnetic ones. Thus, in the remainder of this note only terms with indices γ and γZ are included. The index γ will be dropped, since there is no ambiguity, and the $\eta^{\gamma Z}$ coefficient will be labeled just η. By parity conservation in the electromagnetic interaction, the terms $F_3 = g_3 = g_4 = g_5 = 0$, so they will be dropped too.

It is important to note that the electroweak coefficient $C^{\gamma Z} = g_V - \lambda g_A$ depends on the beam polarization. Thus, it is convenient to expand the sum of eq.(1) into all its components, so that their polarization dependence is explicitly shown

$$
\begin{aligned}
\frac{d^2\sigma^{lN}_{nc}}{dxdy}(\lambda, P_t) = 8\pi MxyE\frac{\alpha^2}{Q^4}\Big\{ & k_1F_1 + \frac{k_2}{x}F_2 - \lambda P_t(k_4g_1 - 2k_5g_2) \\
& +\eta g_V\Big[k_1F_1^{\gamma Z} + \frac{k_2}{x}F_2^{\gamma Z} - \lambda k_3F_3^{\gamma Z} \\
& -\lambda P_t(k_4g_1^{\gamma Z} - 2k_5g_2^{\gamma Z}) + P_t(\frac{k_2}{x}g_3^{\gamma Z} - \frac{k_6}{x}g_4^{\gamma Z} + k_7g_5^{\gamma Z})\Big] \\
& -\lambda\eta g_A\Big[k_1F_1^{\gamma Z} + \frac{k_2}{x}F_2^{\gamma Z} - \lambda k_3F_3^{\gamma Z} \\
& -\lambda P_t(k_4g_1^{\gamma Z} - 2k_5g_2^{\gamma Z}) + P_t(\frac{k_2}{x}g_3^{\gamma Z} - \frac{k_6}{x}g_4^{\gamma Z} + k_7g_5^{\gamma Z})\Big]\Big\}.
\end{aligned}
\tag{3}
$$

It is then easy to see that if the beam and the target are unpolarized, the cross section reduces to

$$\frac{d^2\sigma_{nc}^{lN}}{dxdy}(0,0) = \sigma_0 = 8\pi MxyE\frac{\alpha^2}{Q^4}\Big\{k_1F_1 + \frac{k_2}{x}F_2 + \eta g_V\Big[k_1F_1^{\gamma Z} + \frac{k_2}{x}F_2^{\gamma Z}\Big]\Big\}, \quad (4)$$

where the third and fourth terms are a small, negligible correction to the usual DIS unpolarized cross section at SLAC's kinematics. If this component is factored out, the cross section can be written as

$$\begin{aligned} \frac{d^2\sigma_{nc}^{lN}}{dxdy}(\lambda, P_t) = \sigma_0\Big\{1 - \lambda P_t\frac{1}{\sigma_0}(k_4g_1 - 2k_5g_2) \\ +\frac{\eta}{\sigma_0}g_V\Big[-\lambda k_3F_3^{\gamma Z} \\ -\lambda P_t(k_4g_1^{\gamma Z} - 2k_5g_2^{\gamma Z}) + P_t(\frac{k_2}{x}g_3^{\gamma Z} - \frac{k_6}{x}g_4^{\gamma Z} + k_7g_5^{\gamma Z})\Big] \\ -\frac{\eta}{\sigma_0}\lambda g_A\Big[k_1F_1^{\gamma Z} + \frac{k_2}{x}F_2^{\gamma Z} - \lambda k_3F_3^{\gamma Z} \\ -\lambda P_t(k_4g_1^{\gamma Z} - 2k_5g_2^{\gamma Z}) + P_t(\frac{k_2}{x}g_3^{\gamma Z} - \frac{k_6}{x}g_4^{\gamma Z} + k_7g_5^{\gamma Z})\Big]\Big\}. \quad (5) \end{aligned}$$

The notation can be simplified considerably if this equation is written in terms of asymmetries

$$\begin{aligned} \frac{d^2\sigma_{nc}^{lN}}{dxdy}(\lambda, P_t) = \sigma_0\Big\{1 - \lambda P_tA_\parallel \\ +\eta\Big[g_V\Big(-\lambda A_3^{\gamma Z} - \lambda P_tA_\parallel^{\gamma Z} + P_tA_\parallel'^{\gamma Z}\Big) \\ -g_A\Big(\lambda A_U^{\gamma Z} - \lambda^2A_3^{\gamma Z} - \lambda^2P_tA_\parallel^{\gamma Z} + \lambda P_tA_\parallel'^{\gamma Z}\Big)\Big]\Big\}, \quad (6) \end{aligned}$$

where the asymmetries have been defined by grouping terms in eq.(5) that have the same λ and P_t coefficients. For example $A_3^{\gamma Z} = k_3F_3^{\gamma Z}/\sigma_0$ and $A_U^{\gamma Z} = (1/\sigma_0)(k_1F_1^{\gamma Z} + k_2F_2^{\gamma Z}/x)$, etc. $A_\parallel$ is the desired parallel asymmetry in spin structure studies.

With this notation it is easy to see that for unpolarized targets the cross section is

$$\sigma^{EW}(\lambda) = \sigma_0\Big\{1 + \eta\Big[g_V(-\lambda A_3^{\gamma Z}) - g_A(\lambda A_U^{\gamma Z} - \lambda^2A_3^{\gamma Z})\Big]\Big\}. \quad (7)$$

The electroweak cross section difference for two beam polarization orientations L and R is then

$$\sigma^{EW\ L} - \sigma^{EW\ R} = \Delta\sigma^{EW} = -2P_b\sigma_0\Big[\eta(g_VA_3^{\gamma Z} + g_AA_U^{\gamma Z})\Big], \quad (8)$$

where P_b is the effective beam polarization. The parton model results of ref.[1, 2] for the structure functions can be applied to evaluate the above expression in terms of known quantities. In the parton model $F_2^i = 2xF_1^i$ (an extended Callan-Gross relation). Then

$$\Delta\sigma^{EW} = -2P_b\eta\Big[g_V k_3 F_3^{\gamma Z} + g_A(k_1 F_1^{\gamma Z} + \frac{k_2}{x}F_2^{\gamma Z})\Big]. \tag{9}$$

Using the Callan-Gross relation and expanding the kinematic coefficients one has

$$\Delta\sigma^{EW} = -2P_b\eta\Big[g_V(1-\frac{y}{2})F_3^{\gamma Z} + g_A(y+\frac{2}{y}-2-\frac{xM}{E})F_1^{\gamma Z}\Big] \tag{10}$$

which, after a little algebra, and neglecting the term xM/E at high E becomes

$$\Delta\sigma^{EW} = 2P_b\frac{\eta}{y}\Big[\frac{1}{2}g_V(1-(1-y)^2)F_3^{\gamma Z} - g_A(1+(1-y)^2)F_1^{\gamma Z})\Big]. \tag{11}$$

Substituting the parton model expressions for $F_1^{\gamma Z} = \sum_j Q_j(g_V)_j(q_j+\overline{q}_j)$ and $F_3^{\gamma Z} = 2\sum_j Q_j(g_A)_j(q_j - \overline{q}_j)$, where the index j runs over the active quark flavors ($q_j(x)$, $\overline{q}_j(x)$ are the corresponding parton densities and Q_j the quark charges,) and dividing by $2\sigma_0$ one has the usual expression for the electroweak asymmetry

$$A_{EW} = \frac{\Delta\sigma^{EW}}{2\sigma_0} = -P_b\frac{\eta}{y\sigma_0}\Big[g_V(1-(1-y^2))\sum_j Q_j(g_A)_j(q_j-\overline{q}_j)$$
$$+g_A(1+(1-y^2))\textstyle\sum_j Q_j(g_V)_j(q_j+\overline{q}_j)\Big], \tag{12}$$

as found in the original paper by Cahn and Gilman [3], or in eq.(6.2.7) of [4]. ($(g_{(V,A)})_j = I_3^{jL}$ for left-handed quarks). The above equation can be used in combination with the appropriate parton distributions to compute A_{EW} for the proton and the neutron.

Note that if A_{EW} is written as $\Delta\sigma^{EW}/(\sigma^{EW\ L} + \sigma^{EW\ R})$ the denominator contains an additional term $2\lambda^2\eta g_A A_3^{\gamma Z}$. Also, the terms that compose $A_{EW} = \eta(g_V A_3^{\gamma z} + g_A A_U^{\gamma z})$ can be collected under this label (this is the same A_{EW} as eq. (12) for $P_b = 1$).

For nuclear targets other than the proton, the individual $e-p$ and $e-n$ cross sections need to be considered. σ_0 in (13) needs to be replaced by σ_p^0 and σ_n^0

$$\sigma_{EW}(\lambda)_{p(n)} = \sigma^0_{p(n)}(1-\lambda A_{EW}^{p(n)} + \lambda^2\eta g_A A_3^{\gamma Z p(n)}). \tag{13}$$

$\sigma_{p(n)}$ is shorthand for $d^2\sigma_{nc}^{l-p(n)}/(dxdy)$. The term with λ^2 always cancels in the differences of cross sections and its contribution to the sums of cross sections is suppressed by η, so it will be neglected from now on. Considering only incoherent scattering, for a nucleus of mass number A with z protons and n neutrons we have

$$\begin{aligned} \sigma_A = z\sigma_p + n\sigma_n &= z\sigma_p^0(1-\lambda A_{EW}^p) + n\sigma_n^0(1-\lambda A_{EW}^n) \\ &= z\sigma_p^0 + n\sigma_n^0 - \lambda(z\sigma_p^0 A_{EW}^p + n\sigma_n^0 A_{EW}^n) \\ &= \sigma_A^0(1-\lambda A_{EW}^A), \end{aligned} \tag{14}$$

where $A_{EW}^A = (z\sigma_p^0 A_{EW}^p + n\sigma_n^0 A_{EW}^n)/\sigma_A^0$ has the usual form of nuclear DIS asymmetries in terms of nucleon asymmetries.

For the case of polarized beam and polarized target, application of the parton model to eq.(6) eliminates some terms, and simplifies others. In addition to the naive parton result $g_2^{\gamma Z} = 0$, $g_3^i - g_4^i = 2xg_5^i$ for $i \neq \gamma$. For consistency with the transverse polarization case, g_2^γ will be preserved. The resulting form is

$$\begin{aligned} \frac{d^2\sigma_{nc}^{lN}}{dxdy}(\lambda, P_t) = \sigma_0\Big\{1 - \lambda A_{EW} - \lambda P_t A_\parallel \\ +\eta\Big[g_V(-\lambda P_t A_\parallel^{\gamma Z} + P_t A_\parallel'^{\gamma Z}) + g_A(\lambda^2 P_t A_\parallel^{\gamma Z} - \lambda P_t A_\parallel'^{\gamma Z})\Big]\Big\}, \end{aligned} \tag{15}$$

where $A_\parallel^{\gamma Z} = k_4 g_1^{\gamma Z}/\sigma_0$ and $A_\parallel'^{\gamma Z} = (2k_2 + k_7)g_5^{\gamma Z}/\sigma_0$. The approximation $k_2 \cong k_6$ was made to factor out k_2 from $k_2 g_3 - k_6 g_4$, and the term proportional to λ^2 only has been left out, as indicated earlier, but the $\lambda^2 P_t$ needs to be retained.

2 Experimental considerations

In the ideal world of theory, all combinations of detector counts with perfect values of the polarizations are possible. Thus the differences of counts with opposite beam helicities for each target polarization can in principle be subtracted from each other, leading to nice cancellations. For example, from the difference $\sigma_{L+} - \sigma_{R+} - (\sigma_{L-} - \sigma_{R-})$, where L, R refer to the beam and $+, -$ to the target polarization, all terms in eq.(15) not involving the product λP_t drop out, leaving only $A_\parallel, A_\parallel^{\gamma Z}$ and $A_\parallel'^{\gamma Z}$.

This simple picture is valid only under very restrictive conditions, which include having no contributions from other asymmetries (such as A_{EW}), equal magnitudes for opposite target polarizations, symmetric detector acceptance over all angles relative to the quantization axis, etc. Since these conditions are seldom realized in an experimental setup, it is necessary to derive expressions that take into account these and other aspects, such as the need to combine many runs, each with different values of the polarization P^+_{bi}, P^-_{bi}, P^+_{ti}, P^-_{ti}, beam charge $Q_i^{+L}, Q_i^{-L}, Q_i^{+R}, Q_i^{-R}$ and dead time corrections $d_i^{+L}, d_i^{-L}, d_i^{+R}, d_i^{-R}$, which change from run to run. P^+_{bi} is the beam polarization for the ith run with positive target polarization, etc.

We start with the numbers of counts for given combinations of beam (e.g. P^+_{bi}) and target polarizations (e.g. P^+_{ti}) for a simple target made of only nitrogen and hydrogen (the nitrogen represents all the unpolarized nuclei; corrections for nitrogen polarization need a special treatment, not given here.)

$$\begin{aligned} L_i^+ &= Q_i^{+L} d_i^{+L} \Delta E' \Delta\Omega (N_N \sigma_{Ni}^{+L} + N_H \sigma_{Hi}^{+L}) \\ R_i^+ &= Q_i^{+R} d_i^{+R} \Delta E' \Delta\Omega (N_N \sigma_{Ni}^{+R} + N_H \sigma_{Hi}^{+R}) \\ L_i^- &= Q_i^{-L} d_i^{-L} \Delta E' \Delta\Omega (N_N \sigma_{Ni}^{-L} + N_H \sigma_{Hi}^{-L}) \\ R_i^- &= Q_i^{-R} d_i^{-R} \Delta E' \Delta\Omega (N_N \sigma_{Ni}^{-R} + N_H \sigma_{Hi}^{-R}) \end{aligned} \quad (16)$$

where the subindex i runs over all runs in the experiment.[1] $\Delta E' \Delta\Omega$ represent the detector acceptances, which are common to all runs for a given target and cancel out in the asymmetry, so they will be ignored from now on. N_A ($A = N, H, D$, etc.) are the numbers of scattering centers of each kind in the target. The corresponding $\sigma_A^{L(R)}(e, e')$ cross sections are of two kinds: $\sigma_{Ni}^{+/-L(R)}$ depend only on the beam helicity, which is largely constant over many runs, making the subindex i somewhat superflous. $\sigma_{Hi}^{+/-L(R)}$ depend on both beam and target polarization. Here i reflects the fact that the effective polarized cross section eq. (15) depends on the target polarization which has a significant run dependence.

In order to simplify the analysis, it is convenient to be able to factorize as many common terms as possible. If the dead time corrections $d_i^{(+/-)(L/R)}$ were equal or absent, it would be useful to express the charges as $Q_i^{-L} = \beta Q_i^{+L}$, provided $Q_i^{+L} \cong Q_i^{+R}$, etc. This is not the case, and expressing the d_i's in terms of ratios to one another does not lead to further simplification, so we are forced to keep all factors. The best approach is then, to convert the counts to charge and dead time normalized quantities, on a run by run basis, so on the r.h.s. we only have sums of $N_A \sigma_A$ products

$$\frac{L_i^+}{Q_i^{+L} d_i^{+L}} = N_N \sigma_{Ni}^{+L} + N_H \sigma_{Hi}^{+L}$$

[1] Not all terms in the series need to be present. For example, for runs that had only positive target polarization P^+_{ti}, the corresponding terms L_i^-, R_i^- would be zero, etc.

$$
\begin{aligned}
\frac{R_i^+}{Q_i^{+R} d_i^{+R}} &= N_N \sigma_{Ni}^{+R} + N_H \sigma_{Hi}^{+R} \\
\frac{L_i^-}{Q_i^{-L} d_i^{-L}} &= N_N \sigma_{Ni}^{-L} + N_H \sigma_{Hi}^{-L} \\
\frac{R_i^-}{Q_i^{-R} d_i^{-R}} &= N_N \sigma_{Ni}^{-R} + N_H \sigma_{Hi}^{-R}.
\end{aligned} \tag{17}
$$

The nitrogen cross sections can be written as

$$
\begin{aligned}
\sigma_{Ni}^{+(-)L} &= \sigma_N^0 (1 - P_{bi}^{+(-)L} A_{EW}^N) \\
\sigma_{Ni}^{+(-)R} &= \sigma_N^0 (1 + P_{bi}^{+(-)R} A_{EW}^N),
\end{aligned} \tag{18}
$$

The corresponding hydrogen cross sections for positive target polarization are

$$
\begin{aligned}
\sigma_{Hi}^{+L} = \sigma_H^0 \Big\{ 1 - P_{bi}^+ A_{EW} - P_{bi}^+ P_{ti}^+ A_\parallel + \eta \Big[& g_V (-P_{bi}^+ P_{ti}^+ A_\parallel^{\gamma Z} + P_{ti}^+ A_\parallel'^{\gamma Z}) \\
& + g_A \Big((P_{bi}^+)^2 P_{ti}^+ A_\parallel^{\gamma Z} - P_{bi}^+ P_{ti}^+ A_\parallel'^{\gamma Z} \Big) \Big] \Big\}, \\
\sigma_{Hi}^{+R} = \sigma_H^0 \Big\{ 1 + P_{bi}^+ A_{EW} + P_{bi}^+ P_{ti}^+ A_\parallel + \eta \Big[& g_V (+P_{bi}^+ P_{ti}^+ A_\parallel^{\gamma Z} + P_{ti}^+ A_\parallel'^{\gamma Z}) \\
& + g_A \Big((P_{bi}^+)^2 P_{ti}^+ A_\parallel^{\gamma Z} + P_{bi}^+ P_{ti}^+ A_\parallel'^{\gamma Z} \Big) \Big] \Big\},
\end{aligned} \tag{19}
$$

and for negative target polarization

$$
\begin{aligned}
\sigma_{Hi}^{-L} = \sigma_H^0 \Big\{ 1 - P_{bi}^- A_{EW} + P_{bi}^- P_{ti}^- A_\parallel + \eta \Big[& g_V (+P_{bi}^- P_{ti}^- A_\parallel^{\gamma Z} - P_{ti}^- A_\parallel'^{\gamma Z}) \\
& + g_A \Big(-(P_{bi}^-)^2 P_{ti}^- A_\parallel^{\gamma Z} + P_{bi}^- P_{ti}^- A_\parallel'^{\gamma Z} \Big) \Big] \Big\}, \\
\sigma_{Hi}^{-R} = \sigma_H^0 \Big\{ 1 + P_{bi}^- A_{EW} - P_{bi}^- P_{ti}^- A_\parallel + \eta \Big[& g_V (-P_{bi}^- P_{ti}^- A_\parallel^{\gamma Z} - P_{ti}^- A_\parallel'^{\gamma Z}) \\
& + g_A \Big(-(P_{bi}^-)^2 P_{ti}^- A_\parallel^{\gamma Z} - P_{bi}^- P_{ti}^- A_\parallel'^{\gamma Z} \Big) \Big] \Big\},
\end{aligned} \tag{20}
$$

where use has been made of the facts that

• the beam helicity changes sign for L and R counts. During a given run the magnitude for both signs of the helicity is essentially the same.

• the target polarizations P_{ti}^- are negative relative to P_{ti}^+, although their magnitudes need not be equal.

To isolate $A_\parallel$ requires forming a counts asymmetry that combines both signs of the beam target polarization in a difference of differences[2]

$$
\varepsilon = \frac{L^+ - R^+ - (L^- - R^-)}{L^+ + R^+ + L^- + R^-}. \tag{21}
$$

[2]This counts asymmetry cannot be formed on a run by run basis. An extensive discussion on the experimental limitations in forming counts asymmetries is given in ref.[5].

The combination of the four terms is treated in the Appendix. The next best choice is to form separate ε's for the sets of runs with common target polarization

$$\varepsilon^+ = \frac{L^+ - R^+}{L^+ + R^+}, \qquad \varepsilon^- = \frac{L^- - R^-}{L^- + R^-}. \tag{22}$$

Forming sums over the entire set of runs N^+, N^-, where the superscripts refer to the numbers of runs for each sign of the target polarization $P_t^{+/-}$, for the P_t^+ case we have

$$L^+ - R^+ = \sum_{i=1}^{N^+}\Big(\frac{L_i^+}{Q_i^{+L}d_i^{+L}} - \frac{R_i^+}{Q_i^{+R}d_i^{+R}}\Big),$$

$$L^+ + R^+ = \sum_{i=1}^{N^+}\Big(\frac{L_i^+}{Q_i^{+L}d_i^{+L}} + \frac{R_i^+}{Q_i^{+R}d_i^{+R}}\Big), \quad \text{etc.}$$

most of the unwanted asymmetries cancel out in the numerator, leaving

$$\begin{aligned}
L^+ - R^+ = -2N_N\sigma_N^0 \textstyle\sum_i P_{bi}^+ A_{EW}^N + 2N_H\sigma_H^0\Big\{-\sum_i P_{bi}^+ A_{EW} \\
-\textstyle\sum_i P_{bi}^+P_{ti}^+ A_\parallel + \eta\Big[g_V(-\sum_i P_{bi}^+P_{ti}^+ A_\parallel^{\gamma Z}) + g_A\Big(-\sum_i P_{bi}^+P_{ti}^+ A_\parallel'^{\gamma Z}\Big)\Big]\Big\} \\
= -2\Big\{\textstyle\sum_i P_{bi}^+(N_N\sigma_N^0 A_{EW}^N + N_H\sigma_H^0 A_{EW}) \\
+N_H\sigma_H^0 \textstyle\sum_i P_{bi}^+P_{ti}^+(A_\parallel + A'_{EW})\Big\},
\end{aligned} \tag{23}$$

where $A'_{EW} = \eta(g_V A_\parallel^{\gamma Z} + g_A A_\parallel'^{\gamma Z})$.

For the denominator

$$\begin{aligned}
L^+ + R^+ = 2\textstyle\sum_i N_N\sigma_N^0 \\
+2N_H\sigma_H^0\Big\{\textstyle\sum_i 1 + \eta\Big[g_V\sum_i P_{ti}^+ A_\parallel'^{\gamma Z} + g_A\sum_i (P_{bi}^+)^2 P_{ti}^+ A_\parallel^{\gamma Z}\Big]\Big\} \\
\cong 2(N_N\sigma_N^0 + N_H\sigma_H^0)N^+.
\end{aligned} \tag{24}$$

where $\sum_i^{N+} 1 = N^+$ is the total number of runs with positive target polarization. The term in the denominator with the η factor can be entirely neglected, even if the asymmetries were of order unity. The counts asymmetry is then

$$\begin{aligned}
\varepsilon^+ = -\frac{N_N\sigma_N^0}{N_N\sigma_N^0 + N_H\sigma_H^0} < P_b^+ > A_{EW}^N \\
-\frac{N_H\sigma_H^0}{N_N\sigma_N^0 + N_H\sigma_H^0}\Big[< P_b^+ > A_{EW} + < P_b^+P_t^+ > (A_\parallel + A'_{EW})\Big] \\
= f\Big[< P_b^+ > (A_{EW} + \frac{N_N\sigma_N^0}{N_H\sigma_H^0}A_{EW}^N) + < P_b^+P_t^+ > (A_\parallel + A'_{EW})\Big].
\end{aligned} \tag{25}$$

where the averages come from dividing numerator and denominator by N^+.

This result has several remarkable features:

• The factor $N_H\sigma^0_H/(N_N\sigma^0_N + N_H\sigma^0_H)$ is the dilution factor f.

• The unpolarized electroweak asymmetry A_{EW} does not cancel out, because only one sign of the target polarization is involved. It could cancel out if the sets of runs with positive and negative target polarization are combined, as discussed in the Appendix.

• The above result applies only if all the runs with P_t^+ are combined to form a single global ε^+, not if per-run asymmetries are formed first, and later combined in a weighted average. Extensive discussion of the caveats involved in the latter approach is given in [5].

• If $A^p_{EW}(x) \simeq A^n_{EW}(x)$, then A_{EW} can be factored out in (25) and $\varepsilon =< P_b^+ > A_{EW} + f < P_b^+ P_t^+ > (A_\parallel + A'_{EW})$.

• There is a polarized electroweak asymmetry A'_{EW}, which does not cancel out either, and which, per nucleon, may be as large as the unpolarized one. This A'_{EW} will not cancel out even if runs with positive and negative target polarization are combined, because it is the EW partner of the desired $A_\parallel$. It is associated only with the polarized nucleons, so it is suppressed by the dilution factor, but the polarized nucleons in Li or N also contribute to it.

There is a question of whether A'_{EW} is significant enough to have an effect comparable to that of A_{EW}. Once again we use the results of the parton model for estimating the contribution of A'_{EW}. The structure functions that enter in this asymmetry are $A^{\gamma Z}_\parallel = k_4 g^{\gamma Z}_1/\sigma_0$ and $A'^{\gamma Z}_\parallel = (2k_2 + k_7) g^{\gamma Z}_5 \sigma_0$. According to [2], in addition to the well known result for $g_1 = \sum_j Q_j^2(\Delta q_j + \Delta\overline{q}_j)$, the electroweak functions are $g^{\gamma Z}_1 = \sum_j Q_j (g_V)_j(\Delta q_j + \Delta\overline{q}_j)$ and $g^{\gamma Z}_5 = \sum_j Q_j (g_A)_j (\Delta q_j - \Delta\overline{q}_j)$. g_1 and $g^{\gamma Z}_1$ differ only by factors g_V/Q_j, which are of order unity, so both structure functions have comparable magnitudes. Similarly, assuming $\Delta q_j \gg \Delta\overline{q}_j$[3], $g^{\gamma Z}_1 \simeq g^{\gamma Z}_5$. A valid approximate relation is then (up to a factor η)

$$A'_{EW} \simeq (k_4 + 2k_2 + k_7)\frac{g^{\gamma Z}_1}{\sigma_0} \simeq (k_4 + 2k_2 + k_7)\frac{g_1}{\sigma_0} \simeq \frac{k_4 + 2k_2 + k_7}{k_4} A_\parallel. \quad (26)$$

The dominant coefficient is k_2, as it can be seen from fig. 1, where the three coefficients and their combination are plotted for beam energy 32.3 GeV and 2.75° scattering angle. The combination is especially significant

[3]The strange sea quark polarization $|\Delta s = \int_0^1 \Delta s(x)dx| \leq |-0.1|$, which is indicative of very small $\overline{u}$ and $\overline{d}$ polarizations.

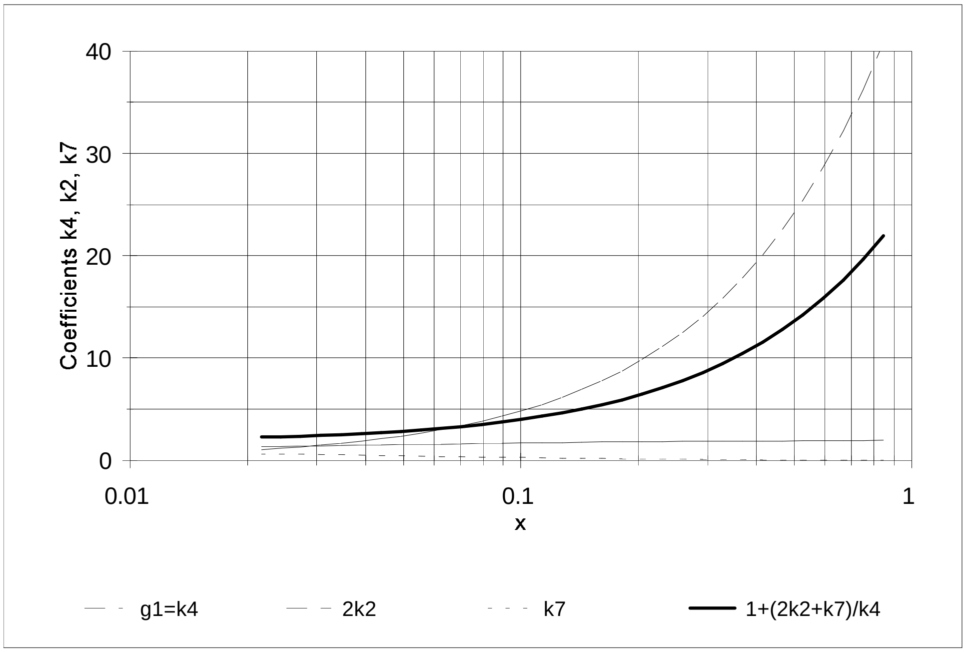

Figure 1: Coefficients k_2, k_4, k_7 and ratio $(k_4 + 2k_2 + k_7)/k_4$, for $E = 32.3$ GeV and $\theta = 2.75°$. The ratio is significant at high x.

in the valence quark region $x >\sim 0.2$. In order to compare the A'_{EW} contribution to that of A_{EW}, in the region where it may be important, we recall that in the valence region $q \gg \overline{q}$, so $F_3^{\gamma Z} \simeq F_1^{\gamma Z}$, and $F_1^{\gamma Z} \simeq F_1$. Ignoring again the common factor η

$$A_{EW} \simeq (k_3 + 2k_2 + k_1)\frac{F_1}{\sigma_0} \simeq \frac{(k_3 + 2k_2 + k_1)}{k_4}\frac{F_1}{g_1}A_{\parallel}. \tag{27}$$

Since $k_3 \simeq k_1 \simeq k_7$, this result indicates that A'_{EW} at $x \geq 0.2$ could be comparable to A_{EW}, largely depending on the actual value of the quark sea polarization. If the sea were to be as polarized as the valence quarks, A'_{EW} would still be a significant fraction of A_{EW}. The ratio between the two asymmetries ranges from

$$\frac{A'_{EW}}{A_{EW}} = \frac{2k_4}{k_3 + 2k_2 + k_1}\frac{g_1}{F_1} \tag{28}$$

to

$$\frac{A'_{EW}}{A_{EW}} = \frac{k_4 + 2k_2 + k_7}{k_3 + 2k_2 + k_1}\frac{g_1}{F_1} \simeq \frac{g_1}{F_1}. \tag{29}$$

However, it is important to recall that A'_{EW} does not cancel when the runs with opposite target polarizations are combined, while A_{EW} cancels out, at least partially . The corrections for A_{EW} and A'_{EW} are very model dependent and require careful analysis that may not be justified given the strong η suppression in longitudinal polarized nucleon scattering at SLAC's kinematics.

3 Transverse Asymmetry

The expression for the transverse cross section, for the case when the polar angle of the nuclear polarization $\alpha = \pi/2$ (in the notation of [2]), is

$$\frac{d^3\sigma^{lN}_{nc}}{dxdyd\phi}(\lambda, P_t) = 4MxyE\frac{\alpha^2}{Q^4}\sum_{i=\gamma.\gamma Z,Z}\eta^i C^i$$
$$\Big\{k_1F_1^i + \frac{k_2}{x}F_2^i - \lambda k_3 F_3^i$$
$$+P_t k_\perp\Big[-\lambda g_1^i - 2\lambda k_4 g_2^i + \frac{k_5}{2x}g_3^i + \frac{k_6}{x}g_4^i - g_5^i\Big]\Big\},$$

and the kinematics coefficients k_m are

$$\begin{aligned} k_1 &= \quad y \\ k_2 &= \quad \frac{1}{y}\Big(1 - y - \frac{xyM}{2E}\Big) \\ k_3 &= \quad 1 - \frac{y}{2} \\ k_4 &= \quad k_5 = \frac{1}{y} \\ k_6 &= \quad \frac{1}{y^2}\Big(1 - y - \frac{xyM}{2E}\Big) \\ k_\perp &= \quad \frac{1}{E}\sqrt{xyM(2(1-y)E - xyM)}\cos(\beta - \phi). \end{aligned} \tag{30}$$

As in the longitudinal case, expanding the C^i coefficients, and using the same conventions, the transverse equivalent of eq.(3) is

$$\frac{d^3\sigma^{lN}_{nc}}{dxdyd\phi}(\lambda, P_t) = 4MxyE\frac{\alpha^2}{Q^4}\Big\{k_1F_1 + \frac{k_2}{x}F_2 - \lambda P_t k_\perp(g_1 + 2k_4 g_2)$$
$$+\eta g_V\Big[k_1 F_1^{\gamma Z} + \frac{k_2}{x}F_2^{\gamma Z} - \lambda k_3 F_3^{\gamma Z}$$
$$+P_t k_\perp(-\lambda g_1^{\gamma Z} - 2\lambda k_4 g_2^{\gamma Z} + \frac{k_5}{2x}g_3^{\gamma Z} + \frac{k_6}{x}g_4^{\gamma Z} - g_5^{\gamma Z})\Big]$$

$$-\lambda\eta g_A\Big[k_1 F_1^{\gamma Z} + \frac{k_2}{x}F_2^{\gamma Z} - \lambda k_3 F_3^{\gamma Z}$$
$$+P_t k_\perp(-\lambda g_1^{\gamma Z} - 2\lambda k_4 g_2^{\gamma Z} + \frac{k_5}{2x}g_3^{\gamma Z} + \frac{k_6}{x}g_4^{\gamma Z} - g_5^{\gamma Z})\Big]\Big\}. \quad (31)$$

The unpolarized cross section is

$$\frac{d^2\sigma_{nc}^{lN}}{dxdyd\phi}(0,0) = \sigma_0 = 4MxyE\frac{\alpha^2}{Q^4}\Big\{k_1 F_1 + \frac{k_2}{x}F_2 + \eta g_V\Big[k_1 F_1^{\gamma Z} + \frac{k_2}{x}F_2^{\gamma Z}\Big]\Big\}, (32)$$

which integrated over ϕ reproduces eq.(4), as expected. In terms of asymmetries

$$\frac{d^3\sigma_{nc}^{lN}}{dxdyd\phi}(\lambda, P_t) = \sigma_0\Big\{1 - \lambda P_t A_\perp$$
$$+\eta\Big[g_V\Big(-\lambda A_3^{\gamma Z} - \lambda P_t A_\perp^{\gamma Z} + P_t A_\perp'^{\gamma Z}\Big)$$
$$-g_A\Big(\lambda A_U^{\gamma Z} - \lambda^2 A_3^{\gamma Z} - \lambda^2 P_t A_\perp^{\gamma Z} + \lambda P_t A_\perp'^{\gamma Z}\Big)\Big]\Big\}, \quad (33)$$

where $A_\perp^i = k_\perp(g_1^i + 2k_4 g_2^i)/\sigma_0$, $A_\perp'^{\gamma Z} = k_\perp(k_5 g_3^{\gamma Z}/(2x) + k_6 g_4^{\gamma Z}/x - g_5^{\gamma Z})/\sigma_0$ and the other asymmetries are as in the longitudinal case.

Clearly, for unpolarized targets

$$\sigma^{EW}(\lambda) = \sigma_0\Big\{1 - \eta\Big[g_V(\lambda A_3^{\gamma Z}) + g_A(\lambda A_U^{\gamma Z} - \lambda^2 A_3^{\gamma Z})\Big]\Big\}, \quad (34)$$

so the contribution of A_{EW} remains unchanged in polarized transverse scattering. Again we will neglect the term proportional to λ^2.

For the experimental counts asymmetries $\varepsilon^{+/-}$, the hydrogen cross sections for positive target polarization are

$$\sigma_{Hi}^{+L} = \sigma_H^0\Big\{1 - P_{bi}^+ A_{EW} - P_{bi}^+ P_{ti}^+ A_\perp + \eta\Big[g_V(-P_{bi}^+ P_{ti}^+ A_\perp^{\gamma Z} + P_{ti}^+ A_\perp'^{\gamma Z})$$
$$+g_A\Big((P_{bi}^+)^2 P_{ti}^+ A_\perp^{\gamma Z} - P_{bi}^+ P_{ti}^+ A_\perp'^{\gamma Z}\Big)\Big]\Big\},$$
$$\sigma_{Hi}^{+R} = \sigma_H^0\Big\{1 + P_{bi}^+ A_{EW} + P_{bi}^+ P_{ti}^+ A_\perp + \eta\Big[g_V(+P_{bi}^+ P_{ti}^+ A_\perp^{\gamma Z} + P_{ti}^+ A_\perp'^{\gamma Z})$$
$$+g_A\Big((P_{bi}^+)^2 P_{ti}^+ A_\perp^{\gamma Z} + P_{bi}^+ P_{ti}^+ A_\perp'^{\gamma Z}\Big)\Big]\Big\}, \quad (35)$$

i.e. identical to the longitudinal case, with the substitution of $A_\perp$ for $A_\parallel$, and similarly for negative target polarization. There is a transverse electroweak asymmetry $(A'_{EW})_\perp = \eta(g_V A_\perp^{\gamma Z} + g_A A_\perp'^{\gamma Z})$, as in eq.(25), which will not cancel even if the target polarization is reversed.

The polarized asymmetry $A_\perp$ is reduced by an extra factor $k_\perp$ compared to $A_\parallel$, since the other coefficients are comparable for the longitudinal and transverse cases

$$\frac{A_\perp}{A_\parallel} = k_\perp \frac{g_1 + 2g_2/y}{(2-y)g_1} \simeq \frac{k_\perp}{2-y}, \tag{36}$$

where the last approximation is valid for $g_1 \gg g_2$. Since $k_\perp \leq 0.05$, this result would indicate that A_{EW} is a much more important component of the measured transverse asymmetry than of the longitudinal one. On the other hand, if the true $g_2 \sim g_1$,

$$\frac{A_\perp}{A_\parallel} \geq \frac{2k_\perp}{y(2-y)}, \tag{37}$$

in which case the electroweak correction is significantly less important, especially for $x > 0.2$.

The contribution of the transverse electroweak asymmetry $(A'_{EW})_\perp$ is much more difficult to estimate than that of the longitudinal one, because the electroweak spin structure functions enter in a combination that does not simplify away the unknown $g_4^{\gamma Z}$, which has a very large associated kinematic coefficient $1232 \geq k_6/x \geq 30$ for $0.02 \leq x \leq 0.85$. Obviously, this coefficient more than compensates for the $k_\perp$ factor, and in fact, could make $(A'_{EW})_\perp$ the dominant asymmetry, depending on the relative size of $g_4^{\gamma Z}$ and of g_1 and g_2.

In the naive parton model $g_2^{\gamma Z} = g_4^{\gamma Z} = 0$. In this model $A_\perp^{\gamma Z} = k_\perp g_1^{\gamma Z}/\sigma_0$ and $A'^{\gamma Z}_\perp = k_\perp(k_5 g_3^{\gamma Z}/(2x) - g_5^{\gamma Z})/\sigma_0 = k_\perp(k_5 - 1)g_5^{\gamma Z}/\sigma_0$, so

$$(A'_{EW})_\perp = \frac{k_\perp}{\sigma_0}\eta\Big(g_V g_1^{\gamma Z} + g_A(k_5 - 1)g_5^{\gamma Z}\Big). \tag{38}$$

Again, using the same approximations as in the longitudinal case, for the valence quark region, up to a factor of η

$$\frac{(A'_{EW})_\perp}{A_\perp} \simeq \frac{(1 + k_5 - 1)g_1}{(1 - k_4)g_1} \simeq \frac{1}{y-1}. \tag{39}$$

where the twist-2 form of $g_2^{WW} = -g_1 + \int_x^1 g_1/t dt$ was used, and the contribution of the integral was taken as $\sim g_1/2$ for $x > 0.2$. With these rather crude approximations it can be seen that there seem to be no large factors cancelling the η suppression of $(A'_{EW})_\perp$ relative to $A_\perp$, so perhaps this correction can be neglected.

4 Conclusion

The final result for the asymmetry (either $A_\parallel$ or $A_\perp$) in terms of ε is

$$A_{\parallel(\perp)} = -(A'_{EW})_{\parallel(\perp)} - \frac{1}{N^+ < P_b^+ P_t^+ > + N^- < P_b^- P_t^- >} \Big[\frac{\varepsilon N_R}{f} + (N^+ < P_b^+ > - N^- < P_b^- >)(A_{EW} + \frac{N_N \sigma_N^0}{N_H \sigma_H^0} A_{EW}^N)\Big], \quad (40)$$

where $N_R = N^+ + N^-$ and the dilution factor f is as defined earlier. This expression reduces to the familiar form $A = \varepsilon/(f P_b P_t)$ for the case $N^+ = N^-$, $P_b^+ = P_b^-$ and $P_t^+ = P_t^-$, if $A'_{EW} = 0$.

Corrections for other unpolarizable nuclear species present in the target can easily be included by adding terms of the form $(N_A \sigma_A^0 / N_H \sigma_H^0) A_{EW}^A$. Corrections for the polarized nucleons in N, ^{6}Li and ^{7}Li are related to $A_\parallel$ (or $A_\perp$) and A'_{EW}. These contributions are suppresed by the ratio of the target polarizations $P_A/P_{H(D)}$, and by the effective nucleon helicity in the nucleus so that, as far as A'_{EW} is concerned, they might be important only for ^{6}Li. The usual N or Li corrections to $A_\parallel$, $A_\perp$ still apply.

If, as discussed earlier, $A_{EW}^p \simeq A_{EW}^n$, then A_{EW} can be factored out and

$$A_{\parallel(\perp)} = -(A'_{EW})_{\parallel(\perp)} - \frac{1}{f(N^+ < P_b^+ P_t^+ > + N^- < P_b^- P_t^- >)} \Big[\varepsilon N_R + (N^+ < P_b^+ > - N^- < P_b^- >) A_{EW}\Big], \quad (41)$$

which shows that A_{EW} is very suppressed, even if the condition on $N^+ = N^-$ is poorly satisfied. For example, for $N^+ = 2N^-$ and $P_b = P_b^+ \sim P_b^-$, there would be a factor of $P_b/3$ suppressing A_{EW}. If $N^+ \simeq N^-$, the factor is even smaller, perhaps making the correction entirely negligible at SLAC's kinematics, even for E155x. Also, it is better to do the correction for A_{EW} after combining the runs for both signs of P_t, to benefit from the cancellation: for an individual run, or even for all runs of the same P_t sign A_{EW} may be significant, but it largely goes away in the combination.

APPENDIX

Here we calculate in detail the right hand side of the full ε for the longitudinal asymmetry:

$$\varepsilon = \frac{L^+ - R^+ - (L^- - R^-)}{L^+ + R^+ + L^- + R^-}. \quad (42)$$

The $L^+ - R^+$ difference is shown in eq.(23), while the combination $L^- - R^-$ is

$$L^- - R^- = -2\Big\{\sum_i P_{bi}^-(N_N\sigma_N^0 A_{EW}^N + N_H\sigma_H^0 A_{EW}) \tag{43}$$

$$-N_H\sigma_H^0 \sum_i P_{bi}^- P_{ti}^-(A_\parallel + A'_{EW})\Big\}. \tag{44}$$

Using as before the notation $< P^{+/-} >= (1/N^{+/-})\sum_i P_i^{+/-}$, where $N^{+/-}$ are the numbers of runs with positive or negative target polarization, the numerator is then

$$\begin{gathered} L^+ - R^+ - (L^- - R^-) = \\ -2N^+\Big\{< P_b^+ > (N_N\sigma_N^0 A_{EW}^N + N_H\sigma_H^0 A_{EW}) \\ + < P_b^+ P_t^+ > N_H\sigma_H^0(A_\parallel + A'_{EW})\Big\} \\ +2N^-\Big\{< P_b^- > (N_N\sigma_N^0 A_{EW}^N + N_H\sigma_H^0 A_{EW}) \\ - < P_b^- P_t^- > N_H\sigma_H^0(A_\parallel + A'_{EW})\Big\} = \\ -2\Big\{[N^+ < P_b^+ > -N^- < P_b^- >](N_N\sigma_N^0 A_{EW}^N + N_H\sigma_H^0 A_{EW}) \\ +[N^+ < P_b^+ P_t^+ > +N^- < P_b^- P_t^- >]N_H\sigma_H^0(A_\parallel + A'_{EW})\Big\}. \end{gathered} \tag{45}$$

Since the average beam polarizations $< P_b^+ >\cong< P_b^- >$, the electroweak asymmetry cancels if $N^+ = N^-$, i.e. if the numbers of runs for each target enhancement are equal, independently of the actual values of the target polarization. However, even if the condition on P_b is satisfied, the polarized electroweak asymmetry A'_{EW} does not cancel.

The denominator is

$$\begin{gathered} L^+ + R^+ + L^- + R^- = 2\Big\{(N_N\sigma_N^0 + N_H\sigma_H^0)(N^+ + N^-) \\ +N_H\sigma_H^0\eta\Big[N^+(g_V < P_t^+ > A_\parallel^{\prime\gamma Z} + g_A < P_b^+ P_t^+ > A_\parallel^{\gamma Z}) \\ -N^-(g_V < P_t^- > A_\parallel^{\prime\gamma Z} + g_A < P_b^- P_t^- > A_\parallel^{\gamma Z})\Big]\Big\} \\ \cong 2(N_N\sigma_N^0 + N_H\sigma_H^0)(N^+ + N^-) \end{gathered} \tag{46}$$

It is easy to see that the term $g_V(N^+ < P_t^+ > -N^- < P_t^- >)A_\parallel^{\prime\gamma Z}$ cancels under the same conditions as the A_{EW} term in the numerator, and the further condition $< P_t^+ >=< P_t^- >$ is needed for cancellation of $A_\parallel^{\gamma Z}$, but the terms with $A_3^{\gamma Z}$ (not shown) would never cancel out, only the factor η suppresses it.

The combination $\varepsilon' = \varepsilon^+ - \varepsilon^-$ is not the same as ε. If the terms with the η coefficient in the denominators $L^+ + R^+$ and $L^- + R^-$ are neglected and $N^+ = N^-$ then $\varepsilon' \cong 2\varepsilon$. An simple average $(\varepsilon^+ + \varepsilon^-)/2$ or even a weighted one, are useless quantities: this combination is equivalent to $L^+ + L^- - (R^+ + R^-)$, and in this case A_{EW} never cancels out and $A_{\|}$ survives only if the magnitudes of the positive and negative target polarization are different.

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