A. Extraction of A_1 and A_2 from two measurements of $A_\parallel$.

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The extraction of $A_1(Q^2,\nu)$ from two measurements of $A_\parallel$ depends on the combination of A_1 and A_2

$$A_1(Q^2,\nu) + \eta(E,E',\theta)A_2(Q^2,\nu) = \frac{A_\parallel(E,E',\theta)}{D(E,E',\theta,R)} \tag{1}$$

so that two measurements of $A_\parallel(E,E',\theta)$ at the (Q^2,ν) intersection of two lines of constant beam energy and scattering angle E_a,θ_a and E_b,θ_b in the Q^2,ν plane can in principle be used to separate A_1 and A_2. E' is the energy of the scattered electron, $\eta = \epsilon\sqrt{Q^2}/(E-\epsilon E')$, $\epsilon^{-1} = 1 + 2[1+(\nu^2/Q^2)]\tan^2(\theta/2)$ is the longitudinal polarization of the virtual photon and $D = (1-\epsilon E'/E/)/(1+\epsilon R)$ is the virtual photon depolarization, with $R = \sigma_L/\sigma_T$.

For

$$A_1(Q^2,\nu) = \frac{1}{\eta_b-\eta_a}\Big(\frac{\eta_b}{D_a}A_\parallel(E_a) - \frac{\eta_a}{D_b}A_\parallel(E_b)\Big)$$

where the a,b subscripts refer to quantities evaluated at the corresponding energies and angles, the uncertainties in $A_\parallel$ are magnified by the factors

$$\delta^2 A_1 = \frac{\eta_b^2}{(\eta_a-\eta_b)^2 D_a^2}\delta^2 A_\parallel(E_a) + \frac{\eta_a^2}{(\eta_a-\eta_b)^2 D_b^2}\delta^2 A_\parallel(E_b). \tag{2}$$

Since both $D < 1$ and $\eta < 1$, the magnification factors (MF's) are greater than 1. The corresponding expressions for A_2 are

$$A_2(Q^2,\nu) = \frac{1}{\eta_a-\eta_b}\Big(\frac{A_\parallel(a)}{D_a} - \frac{A_\parallel(b)}{D_b}\Big) \tag{3}$$

and

$$\delta^2 A_2 = \frac{1}{D_a^2(\eta_a-\eta_b)^2}\delta^2 A_\parallel(a) + \frac{1}{D_b^2(\eta_a-\eta_b)^2}\delta^2 A_\parallel(b). \tag{4}$$

The spin asymmetries are related to the measured asymmetries for the longitudinal and transversal configurations of the beam and target spins by

$$A_1 = \frac{C}{D}\Big(A_{\|} - dA_{\perp}\Big) \tag{5}$$

$$A_2 = \frac{C}{D}\Big(c'A_{\|} + d'A_{\perp}\Big) \tag{6}$$

where $C = 1/(1+\eta c')$, $c' = \eta(1+\epsilon)/(2\epsilon)$, $d' = 1/\sqrt{2\epsilon/(1+\epsilon)}$ and $d = \eta d'$.

Table 1.

W GeV	Q^2 GeV^2	E_a GeV	θ_a	E_b GeV	θ_b	MF_{a1}	MF_{b1}	MF_{a2}	MF_{b2}	MF_1	MF_2
1.23	1.4	5.734	13.15°	2.284	41.7°	7.9	4.3	15.4	5.0	5.2	5.4
1.68	1.226	5.734	13.28°	2.558	42.6°	3.6	2.8	14.2	5.5	3.6	4.5

Table 1 shows the cases of the $P_{33}(1232)$ and the $F_{15}N(1680)$ resonance for two pairs of values of the energies and angles, that match the low Q^2 setting of E96-002. The energies were paired to give the greatest $\Delta\epsilon$ ($\Delta\epsilon = 0.301$ at invariant mass $W = 1.23$ GeV and 0.476 at $W = 1.68$ GeV) that is consistent with the 50° maximum scattering angle allowed by the target geometry. The F_{15} invariant mass cannot be reached at this Q^2 with the same pair of beam energies as the Δ's. The corresponding E96-002 kinematics conditions are 6 GeV beam energy, $\theta = 12.5°$.

The columns labeled $\mathrm{MF}_{a1,b1,etc}$ are the magnification factors that multiply $\delta A_{\|}$ measured alone. For the optimized condition in which both the E_a and E_b data contribute equal uncertainties one has

$$\delta A_{1(2)} = \delta A_{\|}\sqrt{2}MF_{a1(a2)}, \tag{7}$$

with $\delta A_{\|} = 1/(fP_bP_t\sqrt{N})$.

The columns labeled $\mathrm{MF}_{1,2}$ are the factors corresponding to the $A_{\|}, A_{\perp}$ approach (assuming that the time for the perpendicular measurement is one half that of the parallel case). A model independent comparison is thus possible between the two methods. For equal uncertainties in $A_{1(2)}$ one needs

$$\delta A_{\parallel}(E_a, E_b) = \frac{MF_{1(2)}}{\sqrt{2}MF_{a1(a2)}}\delta A_{\parallel}(\parallel, \perp) \tag{8}$$

The expected errors in E96-002 are $\delta A_{\parallel} = 0.007$ and $\delta A_{\perp} = 0.01$ in bins of $\Delta W = 30$ MeV. The corresponding errors in the two $A_{\parallel}$ method for extracting A_1 at $W = 1.23$ GeV need to be $\delta A_{\parallel}(E_a) = 0.0032$ and $\delta A_{\parallel}(E_b) = 0.006$, and for A_2 are $\delta A_{\parallel}(E_a, E_b) = 0.0025$ and 0.008. The statistics needed at each energy are increased correspondingly, as the square of the inverse ratio of eq.(8). The time needed to attain the desired statistics depends on the counting rate at the appropriate kinematics. The large angle measurement will have a much reduced counting rate.

As a practical example, Table 2. displays the expected statistics, rates and times for the examples in table 1, and the additional case of the $\Delta(1232)$ at the same energies as the $N(1680)$ ($\delta\epsilon = 0.226$).

Table 2.

W	Q^2	E_a	θ_a	E_b	θ_b	MF$_a$	MF$_b$	N_a	N_b	R_a	R_b	t_a	t_b
GeV	GeV^2	GeV		GeV				10^6	10^6	Hz	Hz	h	h
1.23	1.4	5.734	13.15°	2.284	41.7°	7.9	4.3	9.2	2.8	4.7	0.81	545	946
1.68	1.226	5.734	13.2°	2.558	43.6°	3.6	2.7	9.3	5.5	10.8	1.5	242	1017
1.23	1.4	5.734	13.15°	2.558	35.45°	10.9	6.1	18	5.6	4.7	1.06	1062	1466

The columns labeled $N_{a,b}$ are the numbers of counts needed to measure A_1 with $\delta A_1 = 0.03$ (0.017), which is the precision expected in E96-002 at the $\Delta(1232)$ ($F_{15}N(1680)$) in a 30 MeV ΔW bin, and which represents a $\sim 12\%$ relative statistical error, for an expected value of $A_1 \sim -0.25$. A dilution factor $f(W)$ of 0.22 at $W = 1.23$ GeV ($f(1.68) = 0.17$), beam polarization P_b of 0.7 and target polarization P_t of 0.8 have been assumed for this estimate.

The columns labeled $R_{a,b}$ are the expected rates for each energy/angle combination, calculated for a 1 cm long $^{15}NH_3$ target with a 60% packing fraction and a 10 nA beam current. The cross sections were computed using Brasse's parameterization of $e-p$ scattering

[1], and the QFS code [2]. The solid angle was estimated as $\Delta\Omega = 2\pi \sin\theta\delta\theta$, with $\delta\theta \sim 46$ mr, corresponding to the Hall C HMS horizontal acceptance used in the E96-002 proposal.

The corresponding MF's for the extraction of A_2 are 15.4 and 5 for the Δ region and 13.8 and 5.3 for the $F_{15}(1680)$. The resulting errors in δA_2 are 0.048 and 0.053, for expected $A_2 \sim 0.1$.

The estimated times for the simultaneous measurement of A_1 and A_2 for the proton in E96-002 are 60 h of $A_{\parallel}$ and 30 h of $A_{\perp}$. (The expected errors in A_2 are 0.04 and 0.03 at each W, for 30 MeV bins). This is to be compared with $\sim$ 2500 h at 10 nA (500 h at 5.7 GeV, and $\sim$ 1000 each at 2.28 GeV and 2.56 GeV, or 1000 h at 5.7 GeV and 1500 h at 2.56 GeV). The range of Q^2 values that can be measured with either pair of energies is limited by the minimum scattering angle at the higher energy (about 8°), so only a few values are possible: The long running times are not compensated by simultaneous measurements of many Q^2 points. Moreover, as the Q^2 decreases, the epsilon range also decreases, with the corresponding growth of the magnification factors. On the other hand, as the Q^2 decreases, the rates improve for the E96-002 method, so that the same number of Q^2points can be measured in much shorter time. In summary, the method has serious limitations compared with the classical parallel-perpendicular separation.

REFERENCES

[1] F. W. Brasse *et al.* , Nucl. Phys. **B110** (1976) 413.

[2] J.W. Lightbody, Jr. and J.S. O'Connell, Comp. Phys., May/June 1998, p. 57.